# HERA Phase I Feed Design 

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#### Abstract

A range of HERA element/feed models are calculated using HFSS in order to determine a preliminary phase I feed design. A number of figures-ofmerit are proposed and a design is presented that optimizes along those parameters. The phase I feed incorporates the existing PAPER dipole and analog signal path (2). The optimized design is a cylindrical structure behind the feed. The backplane is 172 cm in diameter and the cylinder height is 36 cm . The dipole is held off the backplane by a 36 cm mast. The feed is rigged to the antenna from the backplane, and the rigging height is found to be 4.9 m . The design will be revisited in later phases.


## 1 Introduction

The first phase of HERA recycles the PAPER dipole for use in the new feed, in addition to reuse of the complete analog/digital signal path. This memo documents the feed design choice for this first phase. As such, this may be viewed as a preliminary analysis and it will be subsequently revisited.

The feed design is governed by five performance issues:

1. internal reflections
2. cross-talk
3. frequency "smoothness"
4. beam symmetry
5. gain

The next sections discuss the figures-of-merit, followed by the analysis of a wide parameter-space of options and the choice/performance of the implemented design. The feed design optimization seeks to choose a system that balances the design relative to the loosely prioritized list above. The analysis is done assuming no antenna "shroud" (i.e the screen from the primary rim) at this point.

Table 1 summarizes the figures-of-merit.

Table 1: Figure-of-merit Summary

|  | Parameter | f-o-m | f-o-m | f-o-m |
| :--- | :--- | :--- | :--- | :--- |
| 1 | reflections | Feed volume | Feed height | Feed match |
| 2 | cross-talk | Feed efficiency $\epsilon_{f}$ | Feed taper | Feed gain |
| 3 | frequency | Smoothness |  |  |
| 4 | beam | Symmetry $\xi$ | principal axis cuts |  |
| 5 | gain | Mainbeam Efficiency $\epsilon_{b}$ | Feed gain | Feed taper |

## 2 Figures-of-Merit

### 2.1 Internal Reflections

One of the primary specifications for HERA performance is its response in delay space, which requires fairly full and time-consuming modeling. Since the primary issues relate to reflections between the feed and the element, a proxy in this optimization will be to keep the feed volume low. Additionally, low focal heights are preferred to keep reflections at low delay. The feed match also impacts this performance. For all of these parameters, smaller values are better than larger values.

### 2.2 Cross-Talk

Since the HERA dishes are adjacent to one another, there is a concern about cross-talk between them. The figure-of-merit is the feed-efficiency, defined as

$$
\begin{equation*}
\epsilon_{f}=\frac{\int_{-\theta_{e}}^{\theta_{e}} \int_{0}^{\pi} F_{n}(\theta, \phi) \sin \theta d \phi d \theta}{\int_{-\pi}^{\pi} \int_{0}^{\pi} F_{n}(\theta, \phi) \sin \theta d \phi d \theta} \tag{1}
\end{equation*}
$$

where $F_{n}(\theta, \phi)$ is the normalized feed pattern, and $\theta_{e}$ is the feed angle to the edge of the dish

$$
\begin{equation*}
\theta_{e}=2 * \tan ^{-1}\left(\frac{1}{4(f / D)}\right)=76^{o} \tag{2}
\end{equation*}
$$

Large efficiencies are desired.
Additionally, the standard feed taper for the azimuthally-averaged beam is also considered. A large taper is desirable.

### 2.3 Frequency Smoothness/Evolution

Frequency smoothness/evolution is a loose definition such that the overall performance will be chosen to try and lessen its evolution over the entire performance bandwidth. This is done by calculating and comparing the figures-of-merit at five frequencies across the band: $120,137.5,155,172.5,190 \mathrm{MHz}$. Many of the defining performance parameters will be constrained by the lower frequencies. Additionally, many models were also run at 110 MHz .

### 2.4 Beam Symmetry

In order to not introduce a signature due to beam mismatch, it is desired that the beam be as axi-symmetric as feasible. The figure-of-merit is a simplified variant of the fractional power leakage as defined in (1):

$$
\begin{equation*}
\xi=\frac{\int_{-\pi}^{\pi}[\sqrt{G(\theta, \phi=0)}-\sqrt{G(\theta, \phi=90)}]^{2} \sin \theta d \theta}{\int_{-\pi}^{\pi}[\sqrt{G(\theta, \phi=0)}+\sqrt{G(\theta, \phi=90)}]^{2} \sin \theta d \theta} \tag{3}
\end{equation*}
$$

where $G$ is the beam pattern. Similarly, this can be defined for the feed using the feed pattern, $F$. For both, low values (less beam difference) are desired. Additionally, beam-cuts of the principal axes will be examined as well as full sphere models.

### 2.5 Gain

The figure-of-merit for gain (in addition to the gain itself) is the standard beam efficiency

$$
\begin{equation*}
\epsilon_{b}=\frac{\int_{-\theta_{1}}^{\theta_{1}} \int_{0}^{\pi} G(\theta, \phi) \sin \theta d \phi d \theta}{\int_{-\pi}^{\pi} \int_{0}^{\pi} G(\theta, \phi) \sin \theta d \phi d \theta} \tag{4}
\end{equation*}
$$

where $\theta_{1}$ is the main beam half-beamwidth. Normally, this is taken at the first null; however to compare with out-of-focus conditions which fill in that first null, the 20 dB point is also considered. Large values of the beam efficiency (near 1) are desired. For the gain itself, large values at the low frequency end will be favored, since there is intrinsically lower gain at low frequencies.

## 3 EM Modeling

Electromagnetic modeling was done using HFSS. A swept paraboloid was used for simplicity and it was shown earlier that the differences between that and the faceted parabola were not significant (3). Additionally, the conductors are perfect electrical conductors (except for the dipole itself, which was set to be aluminum and copper as appropriate). The dish itself also has a one meter diameter central hole at the vertex. The model drawing is shown in Fig. 1.


Figure 1: HFSS model
The feed considered uses the existing PAPER dipoles (2), with an adjacent metal structure as a "groundplane". Initially, a number of cones versus cylinders were modeled for this structure, however the cone performance was seen to be inferior to the cylinder performance, so the detailed analysis was conducted using cylinders. The cylinders are described by three quantities: the backplane diameter, the cylinder height, and the mast height. Over 200 models of the feed were run and plotted for the five frequencies. The solution frequency was 137.5 MHz . The models ranged over:

- backplane diameter: $56,60,64,68,72$
- cylinder height: $0,4,8,12,16,20,24,28,32,36$
- mast height: $12,14,16,18$


### 3.1 Results

Figure 2 plots outputs all of the feed models, looping first by mast-height ( 4 values), then by cylinder height (10 values), then by cylinder diameter ( 5 values). These plots and the subsequent parameter-space plots form the basis of downselecting feeds for more complete modeling in the full optical system. For each feed configuration, HFSS takes about 20 minutes to compute the performance at those five frequencies. Later a sixth frequency ( 110 MHz ) was added.

Figures 3-6 show the five feed performance metrics across the parameter space, with cylinder height along the Y-axis and cylinder diameter along the Y-axis. Each group of four shows the four mast heights at one of the five frequencies.

Figure 3 shows the feed efficiency. Note that the variation is greater as you decrease frequency, most notably for the cylinder height. Therefore, to increase the efficiency at the low end, short cylinder heights are desirable. It is a weaker function of backplane radius and it is desirable to have smaller feeds from a standing wave point of view, so the diameter will be biased to smaller sizes rather than simply maximizing efficiency.

Figure 4 shows the defined beam asymmetry for the feed with cylinder height along the Y-axis and cylinder diameter along the Y-axis. Each group of four shows the four mast heights at one of the five frequencies.

Figure 5 shows the feed taper for the feed with cylinder height along the Y-axis and cylinder diameter along the Y-axis. Each group of four shows the four mast heights at one of the five frequencies.

Figure 6 shows the feed gain for the feed with cylinder height along the Y-axis and cylinder diameter along the Y-axis. Each group of four shows the four mast heights at one of the five frequencies.


Figure 2: Figures-of-merit of all of the feeds investigated. The large sawtooth in rank order is the backplane diameter variation (56-72). The intermediate sawtooth is the cylinder height (0-36) and the small is the mast height (12-18).


Figure 3: Feed efficiency at 120, 137.5, 155, 172.5, 190 MHz .


Figure 4: Beam asymmetry at 120 MHz .


Figure 5: Feed taper at $120,137.5,155,172.5,190 \mathrm{MHz}$.


Figure 6: Feed gain at 120, 137.5, 155, 172.5, 190 MHz .

### 3.2 Discussion

As mentioned, the faster feed analysis is used to downselect feeds to model in the entire optical path. As a first pass, an efficiency threshold of $90 \%$ is applied to Fig 2, which is shown in Fig. 7. The range includes backplanes with diameters 60,64 and 68 inches. If we key on the 120 MHz results for those backplanes, the efficient cylinder heights are 8,12 and 16 inches, and lower values of mast height (12 and 14) are favored. Also, the efficiency between the peaks of the three backplanes is not great, with slightly less improvement going from 64 to 68 than 60 to 64 .

Looking at that range of designs for the other figures-of-merit, we see that we are near the minimum for polarization mismatch, and maximum for feed taper and gain (for 120 MHz ). So, we will focus on 64 and 68 inch backplanes, 8-16 inch cylinder heights and 12-14 inch mast heights. This smaller range of 10 and 14 inch cylinder heights have also been run. Figure 8 shows the figure-of-merit for this reduced set of parameters. Based on this, the 68 inches was chosen as the backplane diameter and 14 inches as the cylinder height. Note that, given that this will be constructed in South Africa, a change to metric units was desirable. Therefore, the backplane diameter will be specified at 172 cm and the height 36 cm .

The input impedance has also been computed for the various feeds across the band. Figure 9 shows the input reflection coefficient $\left(\mathrm{S}_{11}\right)$ for a number of the models. Clearly 14 " yields better performance generally across the band. The chosen mast height was chosen to be 14in, but again specified as 36 cm . The $172 / 36 / 36 \mathrm{~cm}$ version is then the model carried forward. Figure 10 shows the performance of the 172 cm diameter $/ 36 \mathrm{~cm}$ mastheight performance for various cylinder heights and at the five frequencies.


Figure 7: Zoomed in ordered plots.


Figure 8: Figures-of-merit for the three backplane diameters and four mast heights.


Figure 9: Input reflection coefficient.


Figure 10: Figures-of-merit for 172 cm backplane diameter and 36 cm mast heights at the five frequencies. The black boxes show the chosen configuration.

## 4 Phase Center

The phase center for the chosen design was fit for both E and H planes for $110,120,137.5,155,172.5$, and 190. The fields and fit for the $\mathrm{E} / \mathrm{H}$ modes at 137.5 MHz are shown in Figure 11. The E and H mode fits are shown in Figure 12, as well as the $\mathrm{E} / \mathrm{H}$ and overall averages. The overall average rigging height is 5.12 m . Note that the phase center is actually a region of wavelength scale, so optimization of the rigging height is done by computing the full model as a function around the fitted values.


Figure 11: Illustrative picture at 137.5 MHz for determining the phase center. The black semicircle is a fitted phase front, and the red dot is the fitted phase center. Top plot is the E-plane and bottom is the H-plane.

Figure 13 shows the performance of the chosen feed in the optics, as a function of rigging angle (which corresponds to the height of the backplane). In the top two plots, the heavy black line is the average over frequency. Given that the efficiency average is fairly flat, 4.9 m was chosen as a minimum for the polarization symmetry.

## 5 Beam Models

With the final parameters, a more detailed version was used. This includes additional detail in the feed (stand-offs etc), the central coaxial cable that comes down from the feed, and adding a dielectric approximating dry soil at the hole in the vertex. The following graphics show the principle axis beamcuts along with the azimuthally averaged pattern and the $3-\mathrm{D}$ pattern. And finally, 17 shows some antenna parameters across the $100-200 \mathrm{MHz}$ band. The small jump at 182 MHz is from the first side-lobe merging into the main beam to the calculated resolution of the pattern.


Figure 12: Rigging height for the fitted phase center over frequency. Blue is E-plane (horizontal line is average), green is H-plane (with average) and black is the overall average, which is 5.12 m .

## References

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[3] R. Bradley/M. Venter, Personal communication, NRAO/Univ Stellenbosch, 2015


Figure 13: Performance figures-of-merit of feed for different rigging heights and frequencies.


Figure 14: 110 MHz (left) and 120 MHz (right) principle axis beam cuts with the azimuthallyaveraged and 3-D pattern.


Figure 15: 137.5 MHz (left) and 155 MHz (right) principle axis beam cuts with the azimuthallyaveraged and 3-D pattern.


Figure 16: 190 MHz principle axis beam cuts with the azimuthally-averaged and 3-D pattern.


Figure 17: Plot of various antenna parameters at 1 MHz resolution from $100-200 \mathrm{MHz}$. The solid black line is the theoretical gain at $78 \%$ efficiency.

