LoCo Lab EDGES Memo 158
Antenna Temperature and Beam Chromaticity Correction

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1 Antenna Temperature

The antenna temperature obtained when measuring the sky brightness temperature T_{sky} with an antenna that has a gain pattern G is [1]

$$T_{\mathrm{A}} = \left(\frac{1}{4\pi}\right) \times \int_{\phi=0}^{\phi=2\pi} \int_{\theta=0}^{\theta=\pi} T_{\mathrm{sky}}(\theta,\phi) G(\theta,\phi) \sin\theta d\theta d\phi. \tag{1}$$

2 Computations

When trying to simulate or calibrate sky observations, it is necessary to estimate G. This can be done using EM simulation software packages. However, when including the soil in the simulations, these EM packages normally only provide the fraction of G above the horizon.

A property of G, if the antenna efficiency is 100%, is that its integral over the full sphere is 4π. This property enables us to compute the integrated gain in the soil below the horizon (i.e., the 'ground loss fraction'), which is difficult to get directly from EM simulations, by subtracting from 4π the integrated gain above the horizon.

The mathematical expression is:

$$\begin{aligned} T_{\mathrm{A}} = & \left(\frac{1}{4\pi}\right) \times \int_{\phi=0}^{\phi=2\pi} \int_{\theta=0}^{\theta=\pi/2} T_{\mathrm{sky}}(\theta,\phi) G(\theta,\phi) \sin\theta d\theta d\phi \\ & + T_{\mathrm{soil}} \left[1 - \left(\frac{1}{4\pi}\right) \int_{\phi=0}^{\phi=2\pi} \int_{\theta=0}^{\theta=\pi/2} G(\theta,\phi) \sin\theta d\theta d\phi\right], \end{aligned} \tag{2}$$

where T_{soil} is the physical temperature of the soil.

In practice, if we have the sky and antenna gain models in *Healpix* format (pixels of equal area $\Delta\Omega$), with a number of pixels given by NPixels (for example, for Res=9, NSide=512 and NPixels=$3,145,728$), Equation 2 can be implemented as:

$$T_{\mathrm{A}} = \frac{\sum_{\star} T_{\mathrm{sky}} \cdot G}{\mathrm{NPixels}} + T_{\mathrm{amb}} \left(1 - \frac{\sum_{\star} G}{\mathrm{NPixels}}\right), \tag{3}$$

where $\sum_{\star}$ sums over pixels above the horizon.

[1]Dijk, J., Jeuken, N., & Maanders, E. J. 1968, Proc. IEE, 115, 10.

3 Chromaticity Correction

A correction factor is suggested here to accurately remove the antenna beam chromaticity from sky observations. First, we remove from the measured antenna temperature the effect of ground loss (second term in Equations 2 and 3). Then, we divide by:

$$C(\nu) = \frac{\int_{\phi=0}^{\phi=2\pi} \int_{\theta=0}^{\theta=\pi/2} T_{\mathrm{sky}}(\theta, \phi; \boldsymbol{\nu_0}) G(\theta, \phi; \nu) \sin\theta d\theta d\phi}{\int_{\phi=0}^{\phi=2\pi} \int_{\theta=0}^{\theta=\pi/2} T_{\mathrm{sky}}(\theta, \phi; \boldsymbol{\nu_0}) G(\theta, \phi; \boldsymbol{\nu_0}) \sin\theta d\theta d\phi}. \tag{4}$$

Here, ν_0 is the reference or normalization frequency. Note that we do not normalize the numerator and denominator at each frequency by the solid angle of the beam above the horizon:

$$\Omega_{\mathrm{above}}(\nu) = \int_{\phi=0}^{\phi=2\pi} \int_{\theta=0}^{\theta=\pi/2} G(\theta, \phi; \nu) \sin\theta d\theta d\phi. \tag{5}$$