LoCo Lab EDGES Memo 165
Fisher Matrix Analysis of Physical Foreground Model for EDGES

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1 Description

Here we conduct a Fisher Matrix analysis to estimate the statistical uncertainties in the parameters of the astrophysical foreground (T_0, β, γ, ...) when fitted along with ionospheric opacity (τ_{ion}) and electron temperature (T_e).

The i, j element of the Fisher matrix is given by:

$$F_{ij} = \sum_{n=1}^{N} \frac{1}{[\sigma(\nu_n)]^2} \left[\frac{\partial T}{\partial p_i}(\nu_n) \right] \left[\frac{\partial T}{\partial p_j}(\nu_n) \right], \tag{1}$$

where ν is frequency, T is the model for the brightness temperature spectrum, σ is the standard deviation of the channel noise, N is the number of channels in the spectrum, and p_i is the ith parameter.

To apply a Gaussian prior with standard deviation σ_i^p to the ith parameter, the value of F_{ii} has to be increased by adding $1/(\sigma_i^p)^2$.

The covariance matrix of the parameters is computed as:

$$C = F^{-1}. \tag{2}$$

2 Models

In this memo we compute the parameter uncertainties for the following models:

- Model A (Parameters: T_0, β, γ)

$$T(\nu) = T_{\text{CMB}} + T_0 \left(\frac{\nu}{\nu_0} \right)^{\beta + \gamma \log(\frac{\nu}{\nu_0})} \tag{3}$$

- Model B (Parameters: T_0, β, γ, τ_{ion}, T_e)

$$T(\nu) = T_{\text{CMB}} + T_0 \left(\frac{\nu}{\nu_0} \right)^{\beta + \gamma \log(\frac{\nu}{\nu_0})} e^{-\tau_{ion}\left(\frac{\nu}{\nu_0}\right)^{-2}} + T_e \left[1 - e^{-\tau_{ion}\left(\frac{\nu}{\nu_0}\right)^{-2}} \right] \tag{4}$$

- Model C (Parameters: T_0, β, γ, τ_{ion}, T_e, δ)

$$T(\nu) = T_{\text{CMB}} + T_0 \left(\frac{\nu}{\nu_0} \right)^{\beta + \gamma \log(\frac{\nu}{\nu_0}) + \delta\left[\log(\frac{\nu}{\nu_0})\right]^2} e^{-\tau_{ion}\left(\frac{\nu}{\nu_0}\right)^{-2}} + T_e \left[1 - e^{-\tau_{ion}\left(\frac{\nu}{\nu_0}\right)^{-2}} \right] \tag{5}$$

We use a frequency range $60 - 120$ MHz, a channel width of 0.4 MHz, and a reference frequency $\nu_0 =$ 90 MHz.

3 Nominal Parameter Values

- $T_0 = 1000$ K
- $\beta = -2.6$
- $\gamma = 0.1$
- $T_e = 800$ K
- $\tau_{ion} = 0.005$
- $\delta = -0.2$

4 Noise

We use the following model for the noise standard deviation (Gaussian distribution for each channel, no channel-to-channel correlation):

$$\sigma(\nu) = \sigma_0 \left(\frac{\nu}{\nu_0}\right)^\beta . \tag{6}$$

We use two noise levels at $\nu_0 = 90$ MHz:

- $\sigma_0 = 200$ mK (to mimick a short integration)
- $\sigma_0 = 20$ mK (to mimick a long integration)

5 Priors

In the cases where we use (Gaussian) priors, we use the following priors simultaneously:

- $T_0 : \sigma^p = 500$ K
- $T_e : \sigma^p = 400$ K

We do not use priors for any of the other parameters.

6 Cases

CASE	Model	Noise	Priors
1	A	200 mK	NO
2	A	20 mK	NO
3	B	200 mK	NO
4	B	200 mK	YES
5	B	20 mK	NO
6	B	20 mK	YES
7	C	200 mK	NO
8	C	200 mK	YES
9	C	20 mK	NO
10	C	20 mK	YES

7 Results

The results are presented in Figures 1 through 10.

8 Discussion

1. In Fisher Matrix analyses, it is assumed that the models describe the data perfectly. The uncertainties reported are only statistical.

2. Because of the linearization involved, the uncertainty ellipses are only correct to first order. The true uncertainty distributions in general are not perfectly elliptical.

3. In the figures of this memo we see that for Model A, the parameter uncertainties are very small compared to the cases with more parameters.

4. In Model B, fitting simultaneously the reference astrophysical sky temperature (T_0) and the ionospheric electron temperature (T_e) without priors is not practically possible due to their strong covariance. However, using very conservative priors on these two parameters reduces the uncertainty in T_0 to within a few Kelvin. Although the uncertainty in T_e remains large, it is also reduced. The uncertainty in β, γ, and τ_{ion} is also significantly reduced when priors are used.

5. In Model C (with 6 free parameters) and with 200-mK noise at 90 MHz, the uncertainties are large even with priors. In the case of 20-mK noise (and priors) the uncertainties become reasonably low.

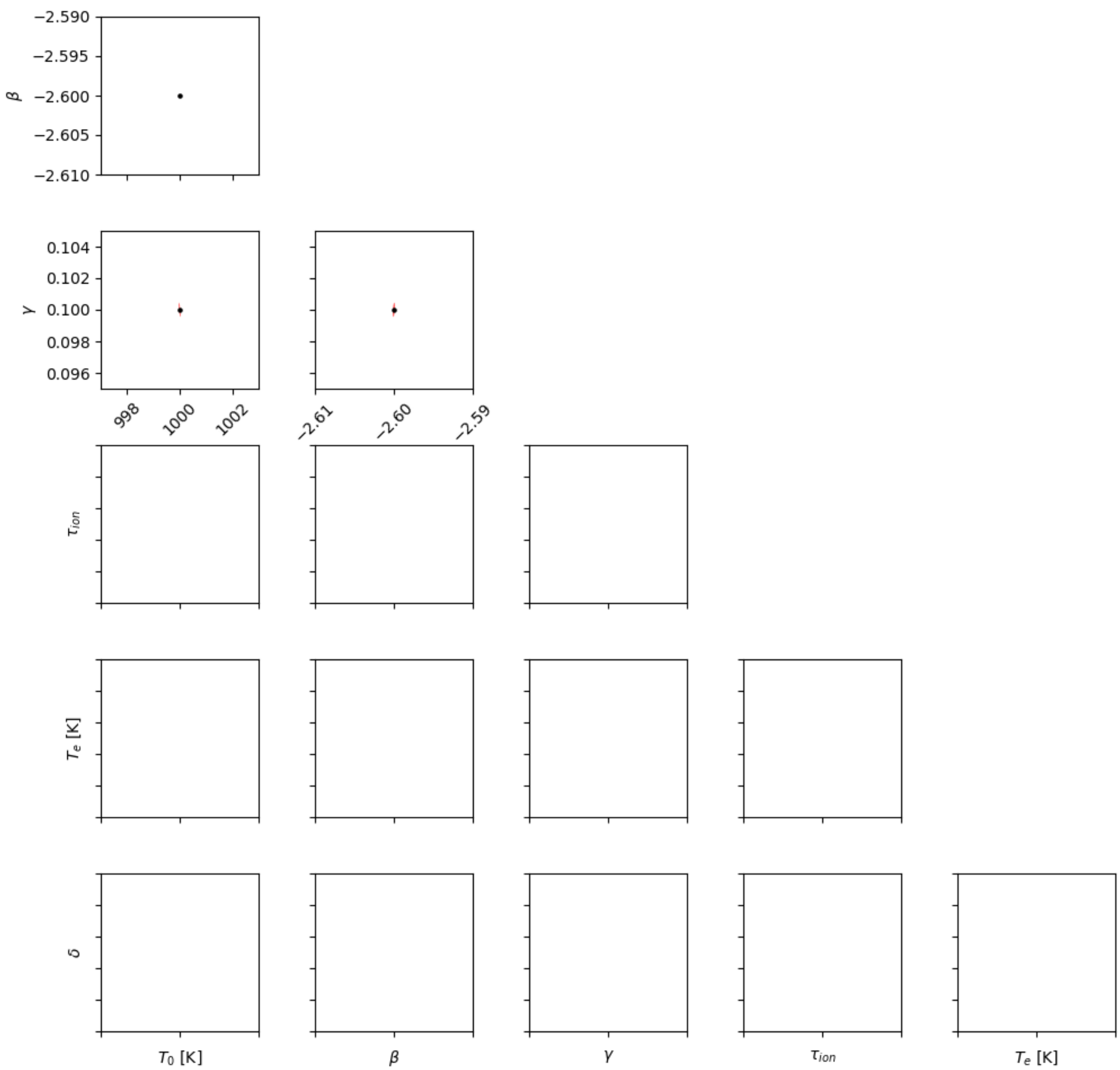

Figure 1: CASE 1. Black dots are the input values. Ellipses correspond to 68% confidence region.

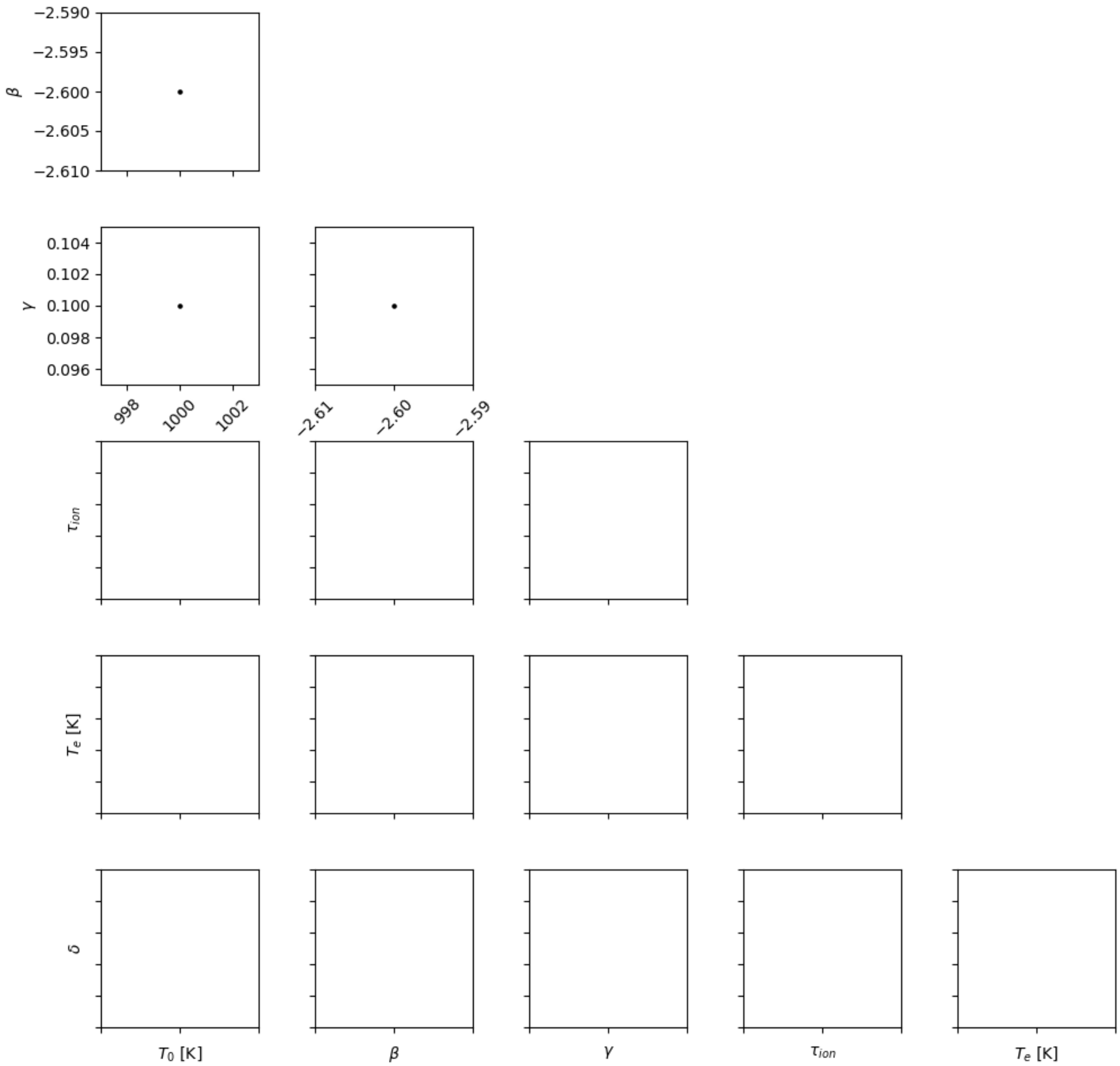

Figure 2: CASE 2. Black dots are the input values. Ellipses correspond to 68% confidence region.

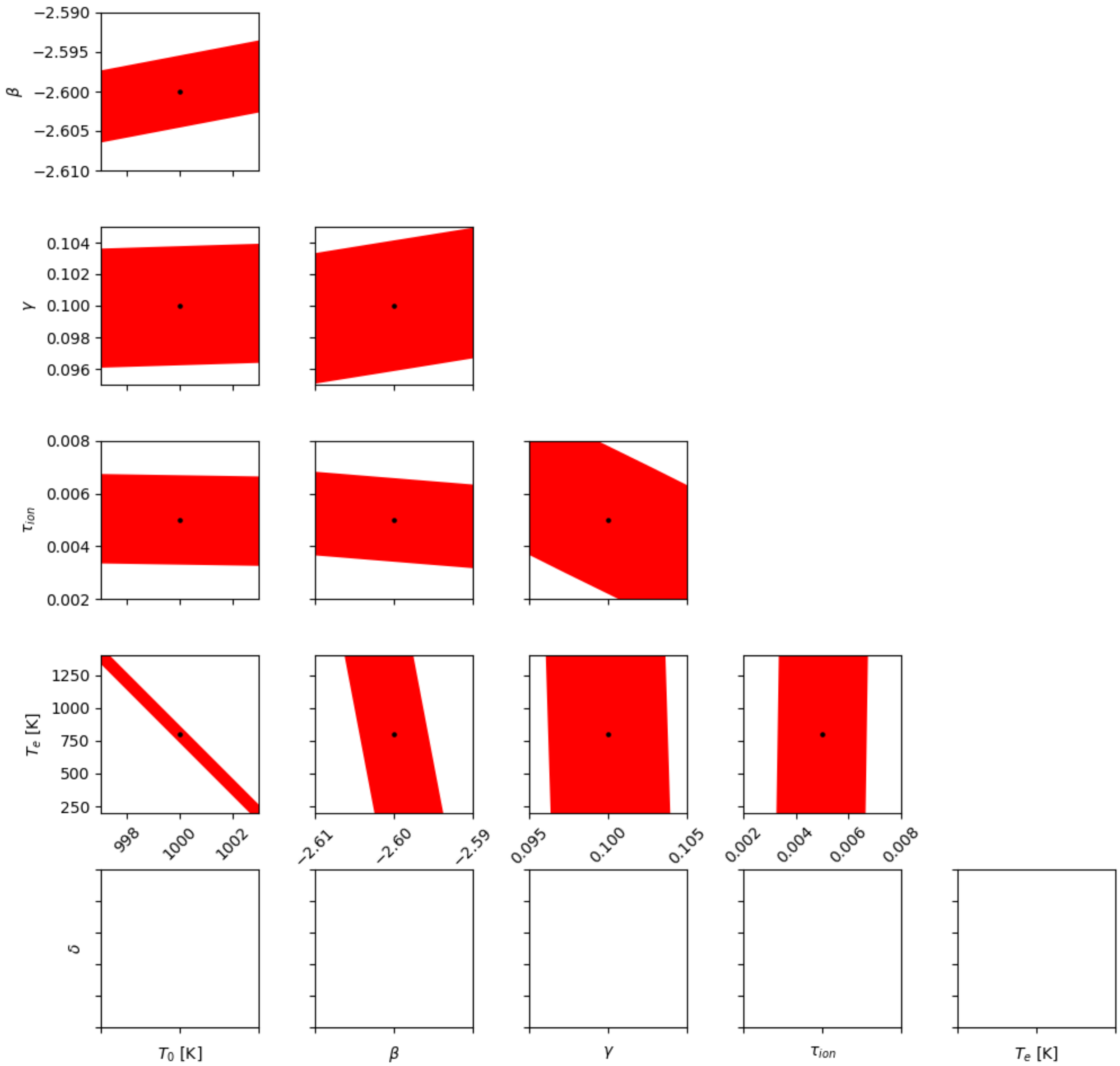

Figure 3: CASE 3. Black dots are the input values. Ellipses correspond to 68% confidence region.

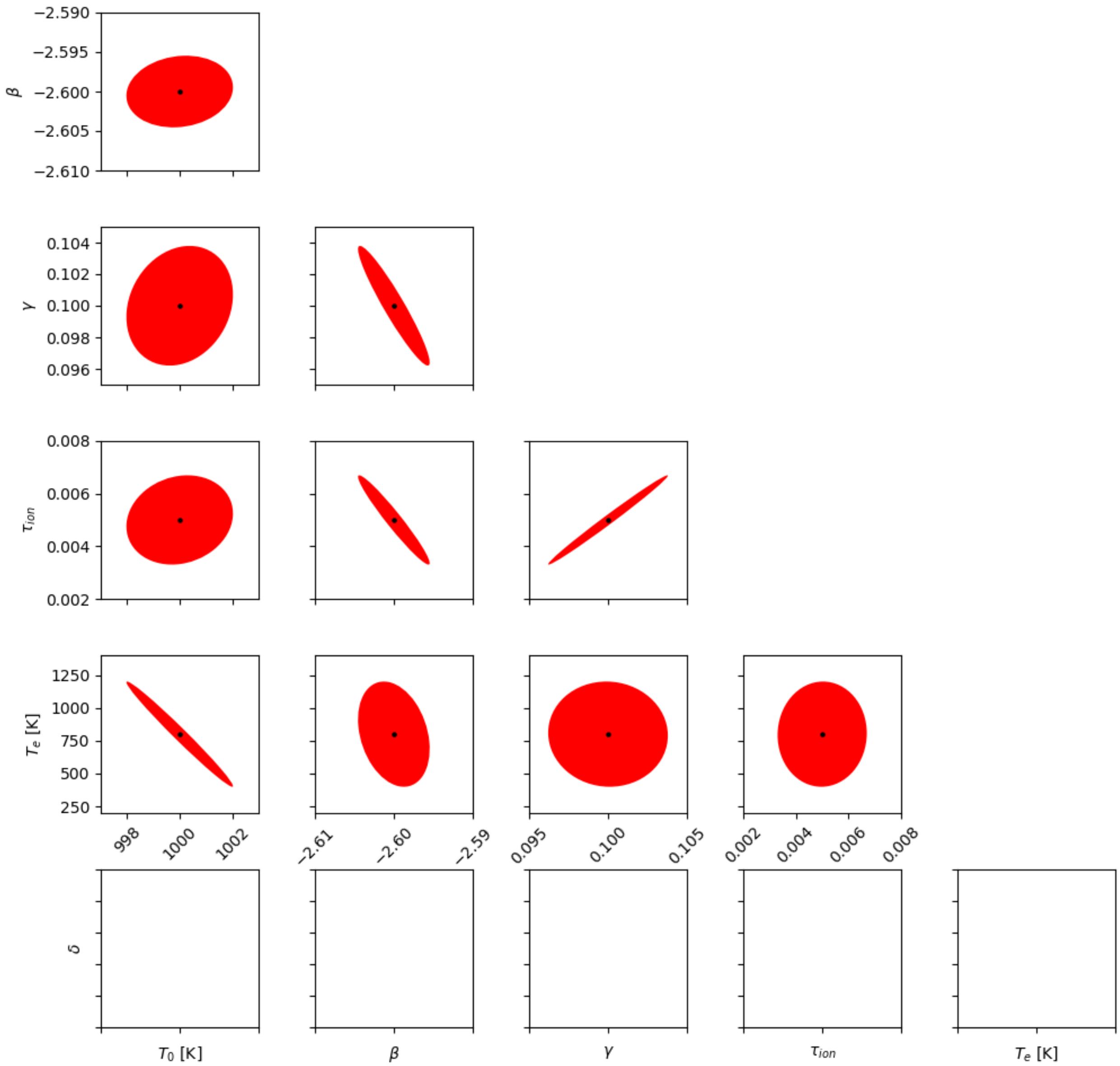

Figure 4: CASE 4. Black dots are the input values. Ellipses correspond to 68% confidence region.

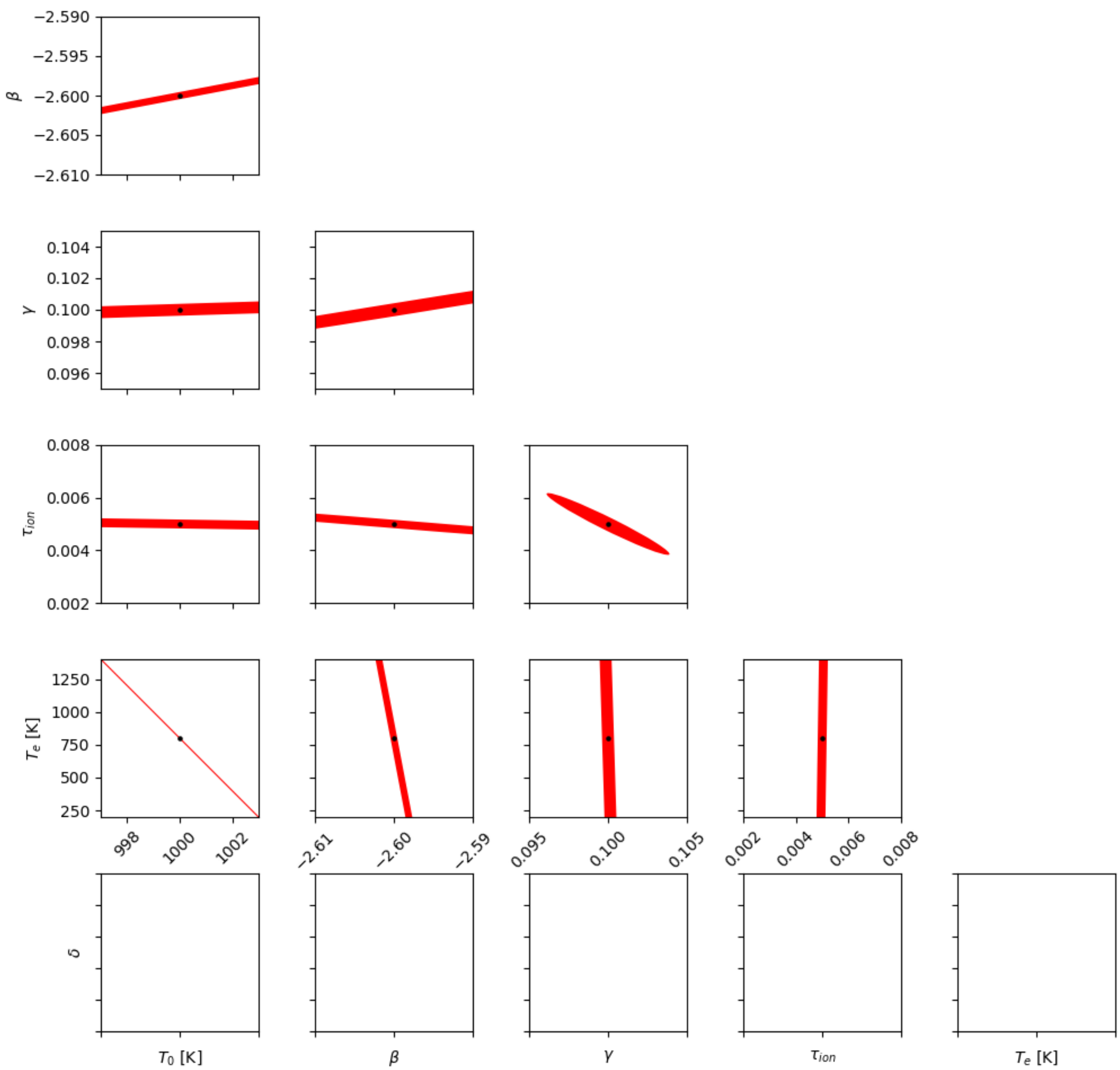

Figure 5: CASE 5. Black dots are the input values. Ellipses correspond to 68% confidence region.

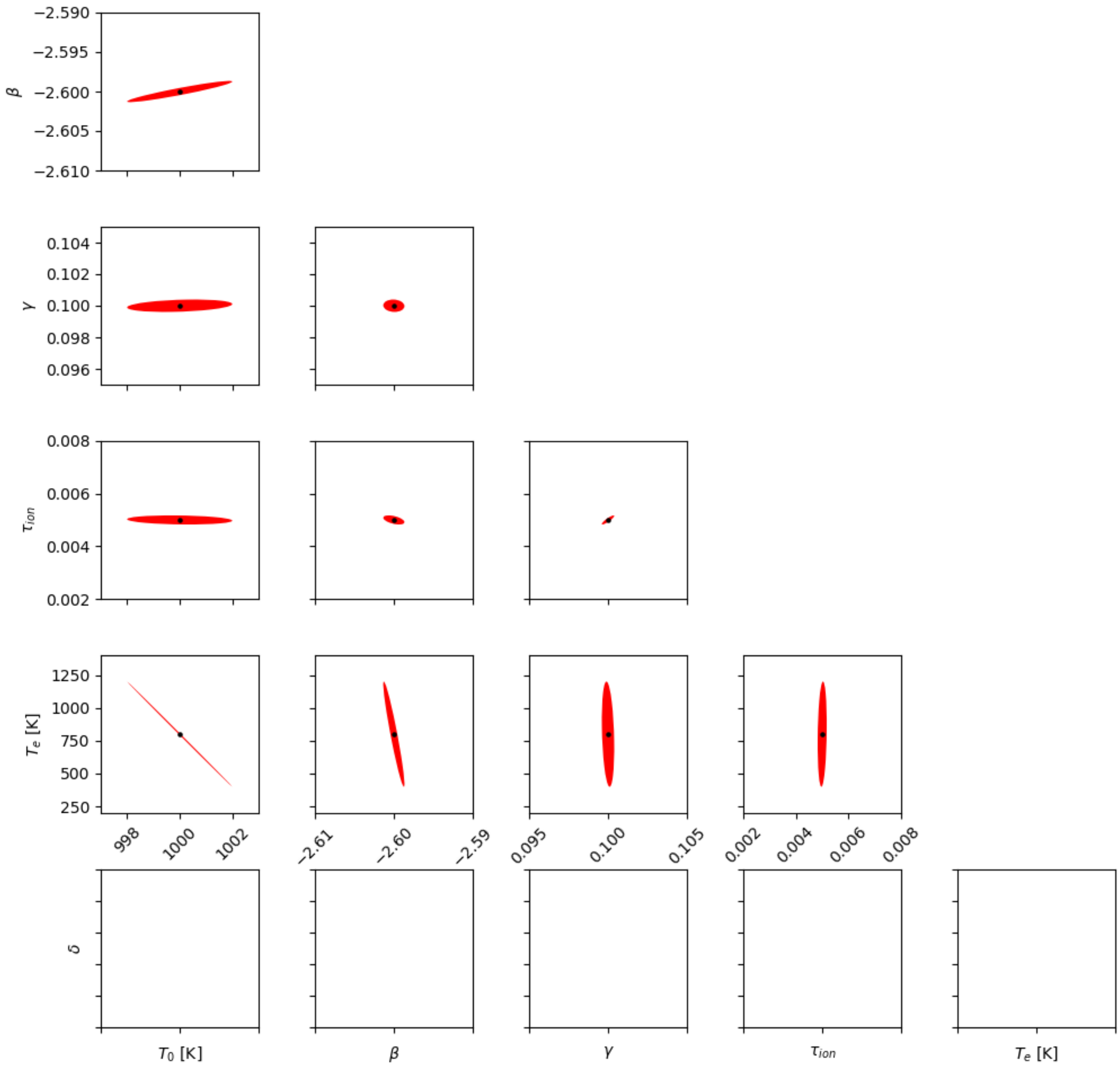

Figure 6: CASE 6. Black dots are the input values. Ellipses correspond to 68% confidence region.

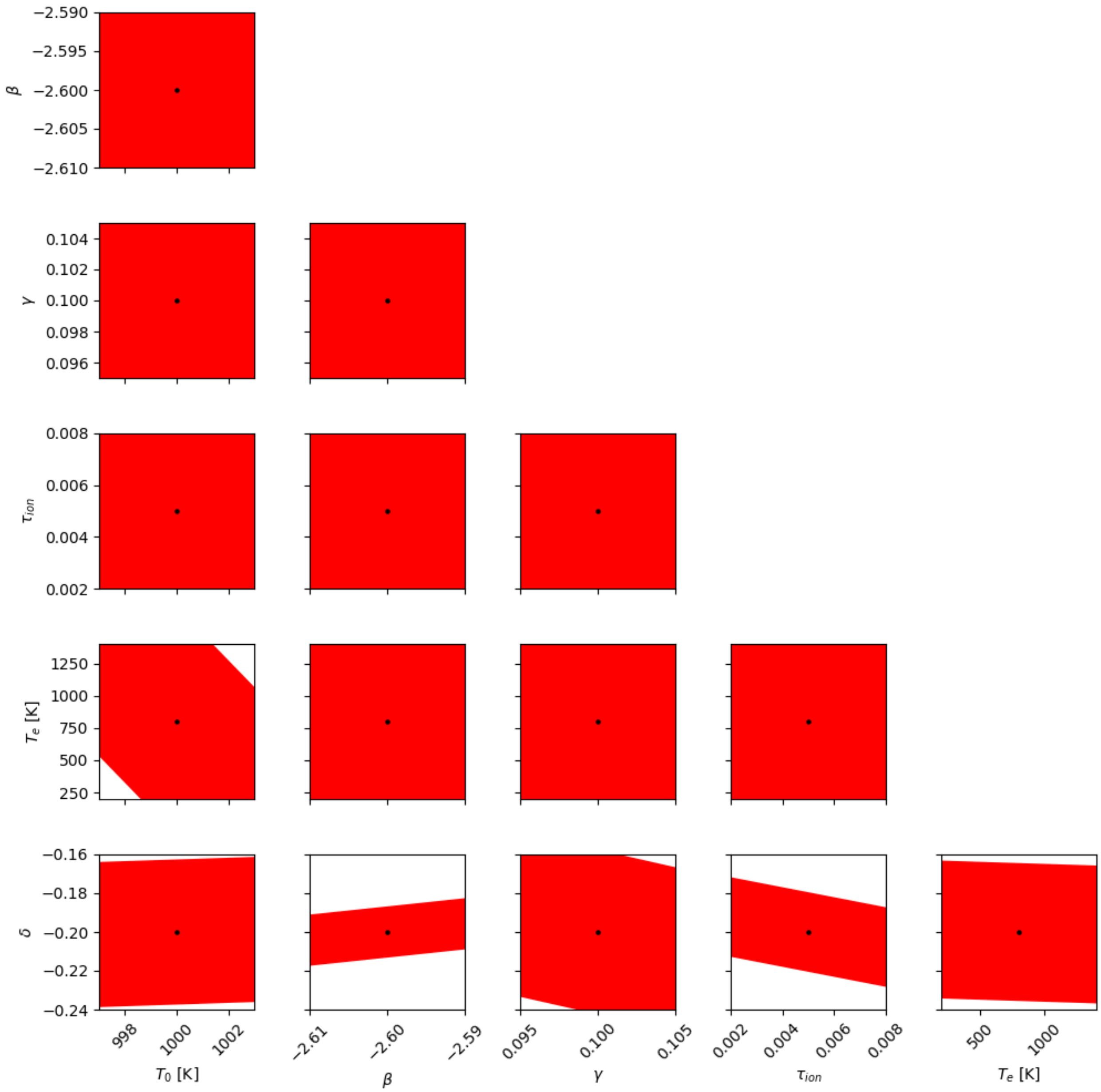

Figure 7: CASE 7. Black dots are the input values. Ellipses correspond to 68% confidence region.

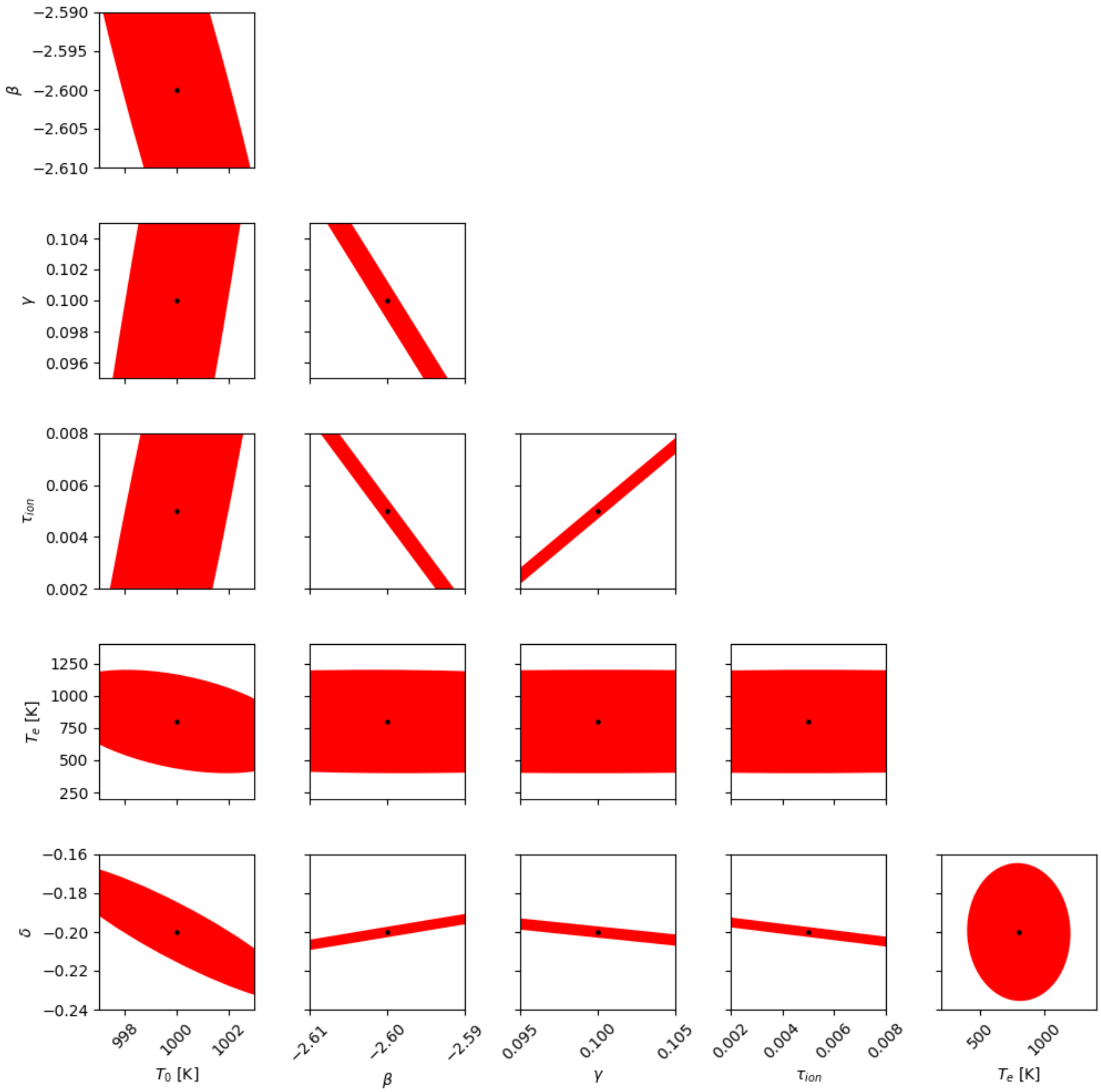

Figure 8: CASE 8. Black dots are the input values. Ellipses correspond to 68% confidence region.

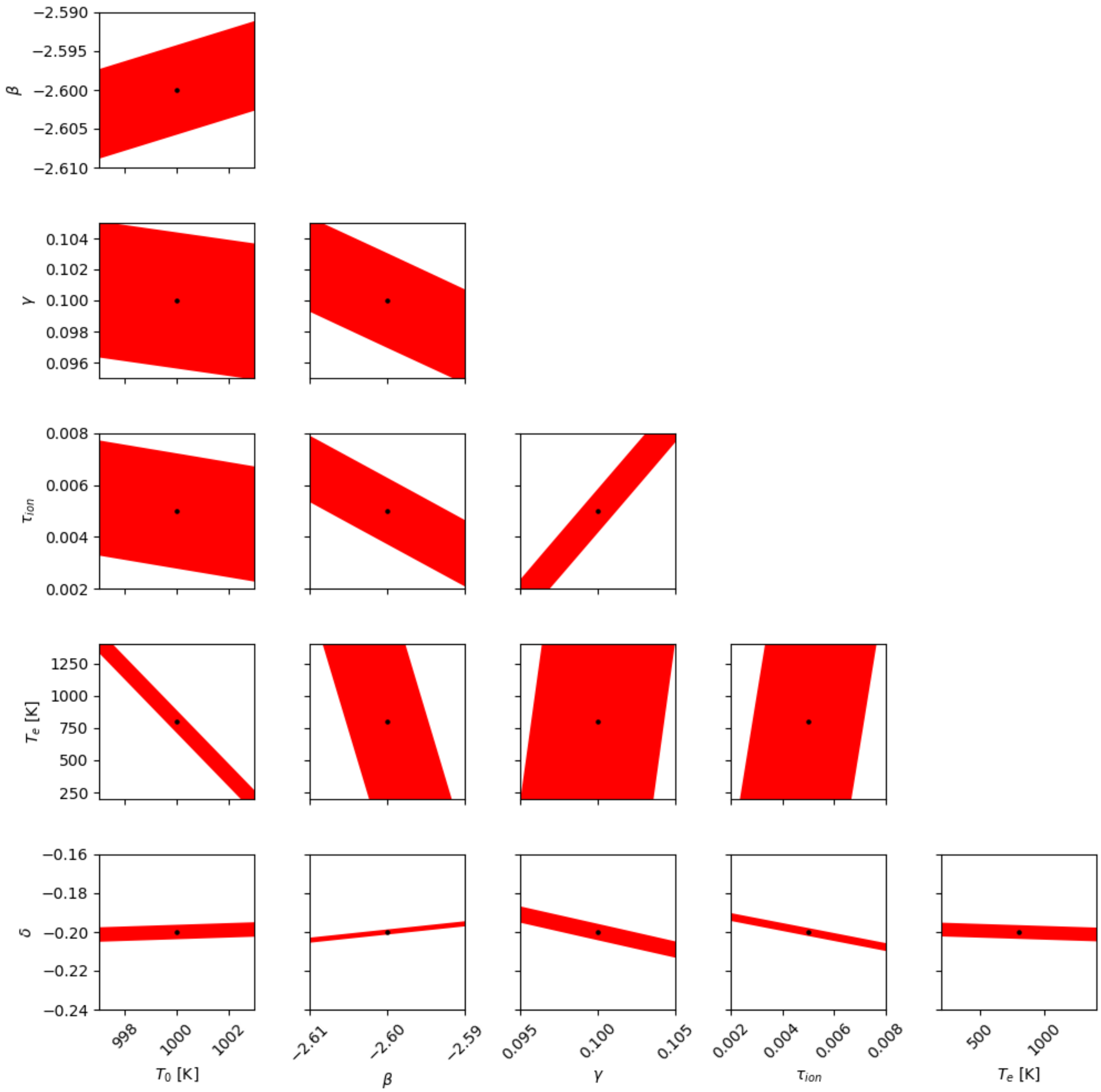

Figure 9: CASE 9. Black dots are the input values. Ellipses correspond to 68% confidence region.

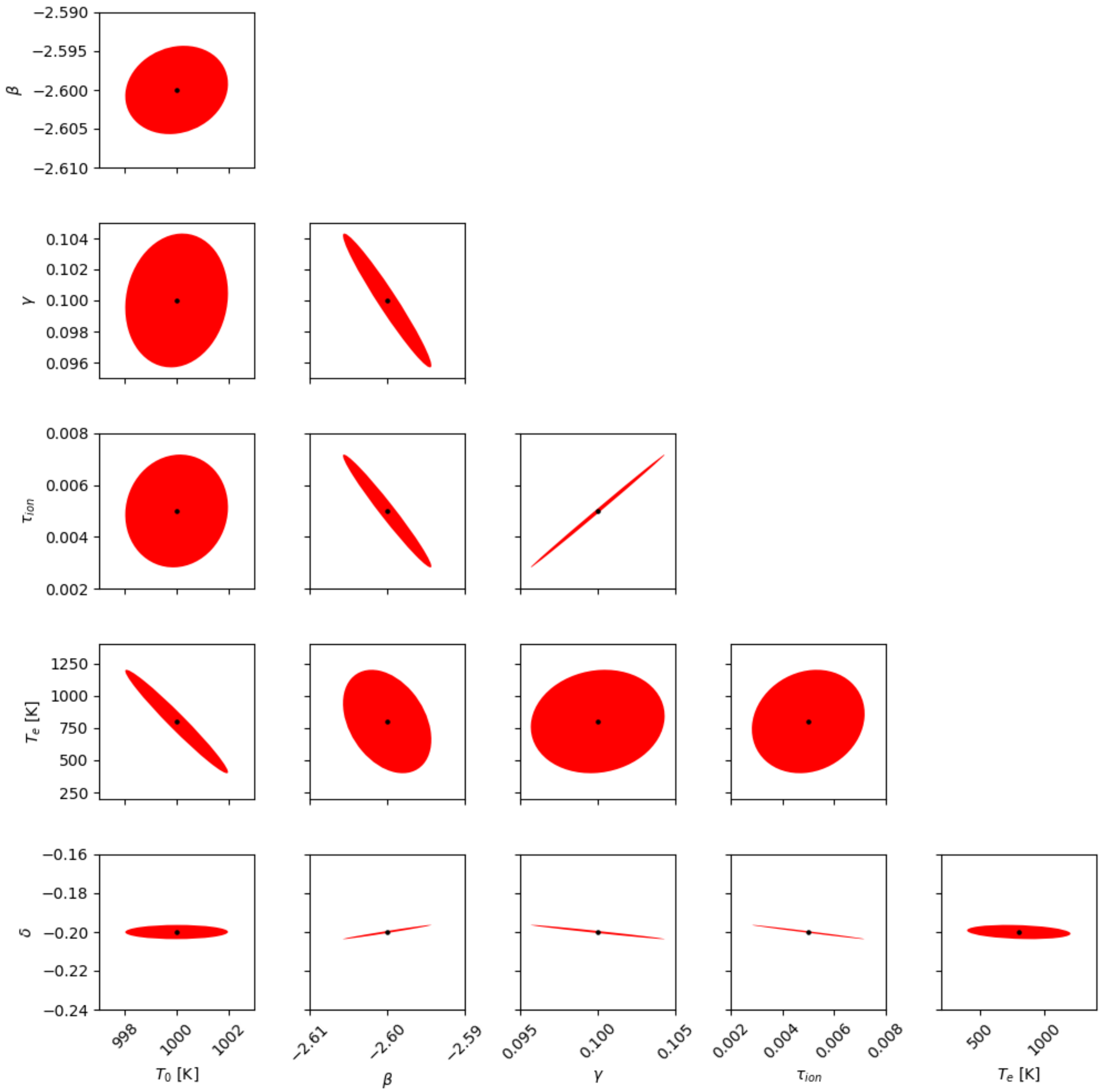

Figure 10: CASE 10. Black dots are the input values. Ellipses correspond to 68% confidence region.