LoCo Lab EDGES Memo 166
Fitting Physical Foreground Model and Flattened Gaussian to Simulated Spectra with PolyChord

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1 Description

Here we fit the 'physical foreground model' — which includes terms for the astrophysical foreground and the ionosphere — and the 'flattened Gaussian' absorption model to simulated spectra using the PolyChord Nested Sampling algorithm. The purpose is to evaluate the convergence of the parameter probability distributions to reasonable ranges given the significant degeneracy that exists between the brightness temperature of the astrophysical foreground at the reference frequency and the ionospheric electron temperature.

This memo continues the exploration of the physical foreground model started in LoCo memo 165.

2 Models

We use the following models:

- Model A (Parameters: T_0, β, γ, τ_{ion}, T_e)

$$T(\nu) = \left[T_{\rm CMB} + T_0\left(\frac{\nu}{\nu_0}\right)^{\beta+\gamma\log(\frac{\nu}{\nu_0})}\right]e^{-\tau_{ion}\left(\frac{\nu}{\nu_0}\right)^{-2}} + T_e\left[1 - e^{-\tau_{ion}\left(\frac{\nu}{\nu_0}\right)^{-2}}\right] \tag{1}$$

- Model B (Parameters: T_0, β, γ, τ_{ion}, T_e, A, ν_0, w, τ)

$$T(\nu) = \text{Flattened Gaussian} + \left[T_{\rm CMB} + T_0\left(\frac{\nu}{\nu_0}\right)^{\beta+\gamma\log(\frac{\nu}{\nu_0})}\right]e^{-\tau_{ion}\left(\frac{\nu}{\nu_0}\right)^{-2}} + T_e\left[1 - e^{-\tau_{ion}\left(\frac{\nu}{\nu_0}\right)^{-2}}\right] \tag{2}$$

- Model C (Parameters: T_0, β, γ, δ, τ_{ion}, T_e, A, ν_0, w, τ)

$$T(\nu) = \text{FlattenedGaussian} + \left[T_{\rm CMB} + T_0\left(\frac{\nu}{\nu_0}\right)^{\beta+\gamma\log(\frac{\nu}{\nu_0})+\delta\left[\log(\frac{\nu}{\nu_0})\right]^2}\right]e^{-\tau_{ion}\left(\frac{\nu}{\nu_0}\right)^{-2}} + T_e\left[1 - e^{-\tau_{ion}\left(\frac{\nu}{\nu_0}\right)^{-2}}\right] \tag{3}$$

We use a frequency range $60-120$ MHz, a channel width of 0.39 MHz, and a reference frequency $\nu_0 = 90$ MHz.

3 Input Parameter Values

- $T_0 = 1000$ K
- $\beta = -2.6$
- $\gamma = 0.1$
- $\delta = 0.2$
- $T_e = 800$ K
- $\tau_{ion} = 0.005$
- $A = -0.5$ K
- $\nu_0 = 78$ MHz
- $w = 19$ MHz
- $\tau = 7$

4 Noise

We use the following model for the noise standard deviation (Gaussian distribution for each channel, no channel-to-channel correlation):

$$\sigma(\nu) = \sigma_0 \left(\frac{\nu}{\nu_0} \right)^{\beta} . \tag{4}$$

We use two noise levels at $\nu_0 = 90$ MHz:

- $\sigma_0 = 200$ mK (to mimick a short integration)
- $\sigma_0 = 20$ mK (to mimick a long integration)

5 Fit Priors

Nominally we use the following very 'Relaxed' uniform priors:

- $T_0 = [-10^4, 10^4]$ K
- $\beta = [-3, -2]$
- $\gamma = [-10^4, 10^4]$
- $\delta = [-10^4, 10^4]$
- $T_e = [0, 10^4]$ K
- $\tau_{ion} = [0, 1]$
- $A = [-2, 2]$ K
- $\nu_0 = [58, 128]$ MHz
- $w = [2, 70]$ MHz
- $\tau = [0.01, 20]$

In some cases we change to 'Tight' uniform priors for T_0 and T_e:

- $T_0 = [500, 1500]$ K
- $T_e = [400, 1200]$ K

6 Cases

CASE	Model	Noise	Priors
1	A	200 mK	Relaxed
2	A	200 mK	Tight
3	A	20 mK	Relaxed
4	A	20 mK	Tight
5	B	20 mK	Relaxed
6	B	20 mK	Tight
7	C	20 mK	Tight

7 Results

The results are presented in Figures 1 through 7.

8 Discussion

1. The main take away of this memo is that fitting a 'physical foreground model' to measurements is feasable under a wide range of noise levels, priors, and number of fit parameters.

2. With the exception of T_e, the model parameters were estimated with relatively good precision and small or no biases in all cases.

3. Applying a tighter prior on T_e can lead to a significant increase in the precision of the estimates, as can be seen by comparing Cases 3 and 4. However, sometimes the precision is good even with relaxed priors on T_e, which can be seen in the similar precision of Cases 5 and 6, which have relaxed and tight priors respectively. In these models, the impact of the prior on the estimates seems to depend on the specific realization of noise in the data.

4. Cases 5, 6, and 7 show that with low measurement noise it is possible simultaneously estimate with precision the parameters of the astrophysical foreground, the ionosphere, and the flattened Gaussian. In particular, in Case 7 (in which we apply a tight prior on T_e) we add δ as a free parameter, for a total of 10, and the precision of the estimates is still good.

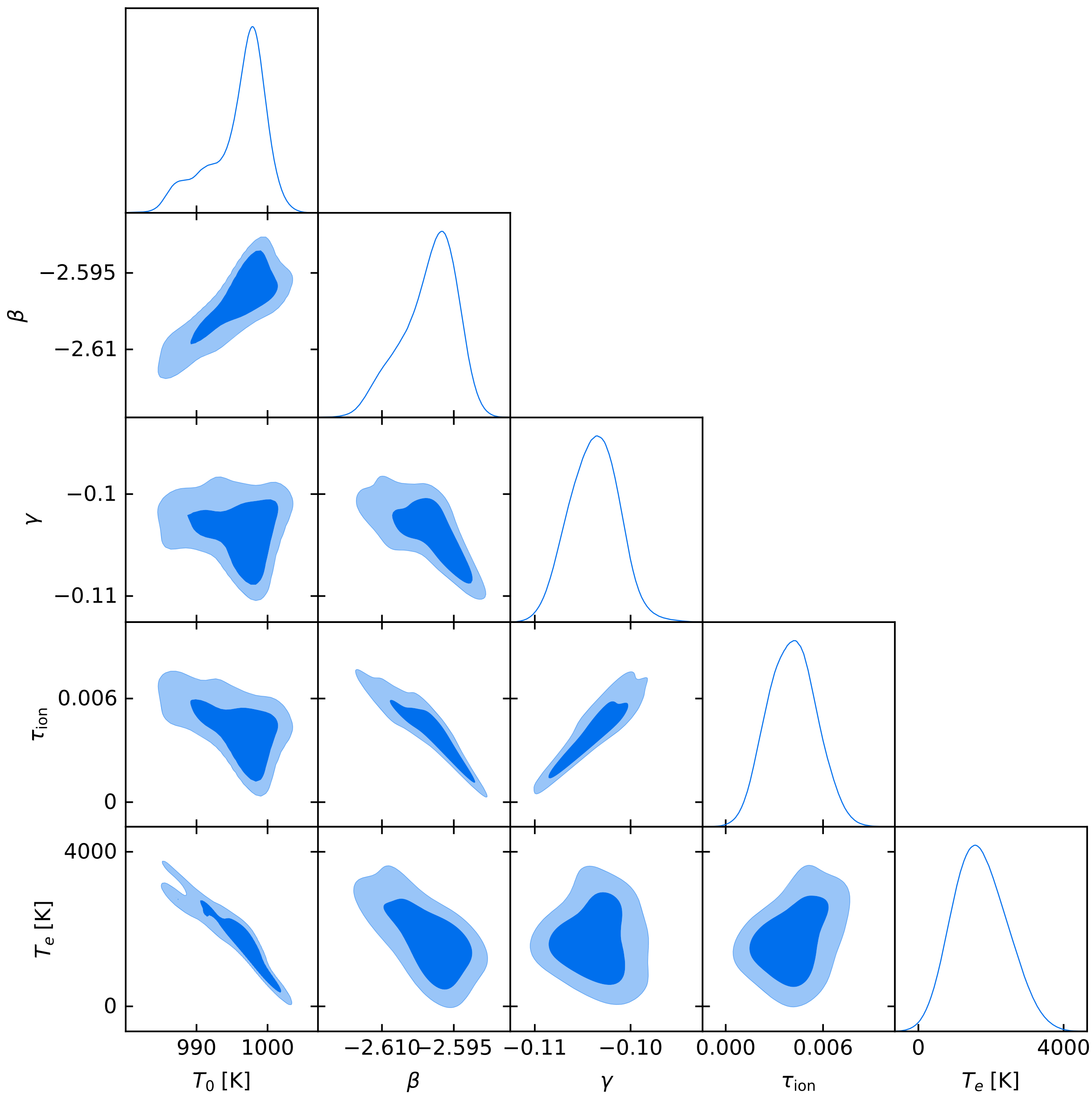

Figure 1: CASE 1. colors correspond to 68% and 95% confidence regions respectively.

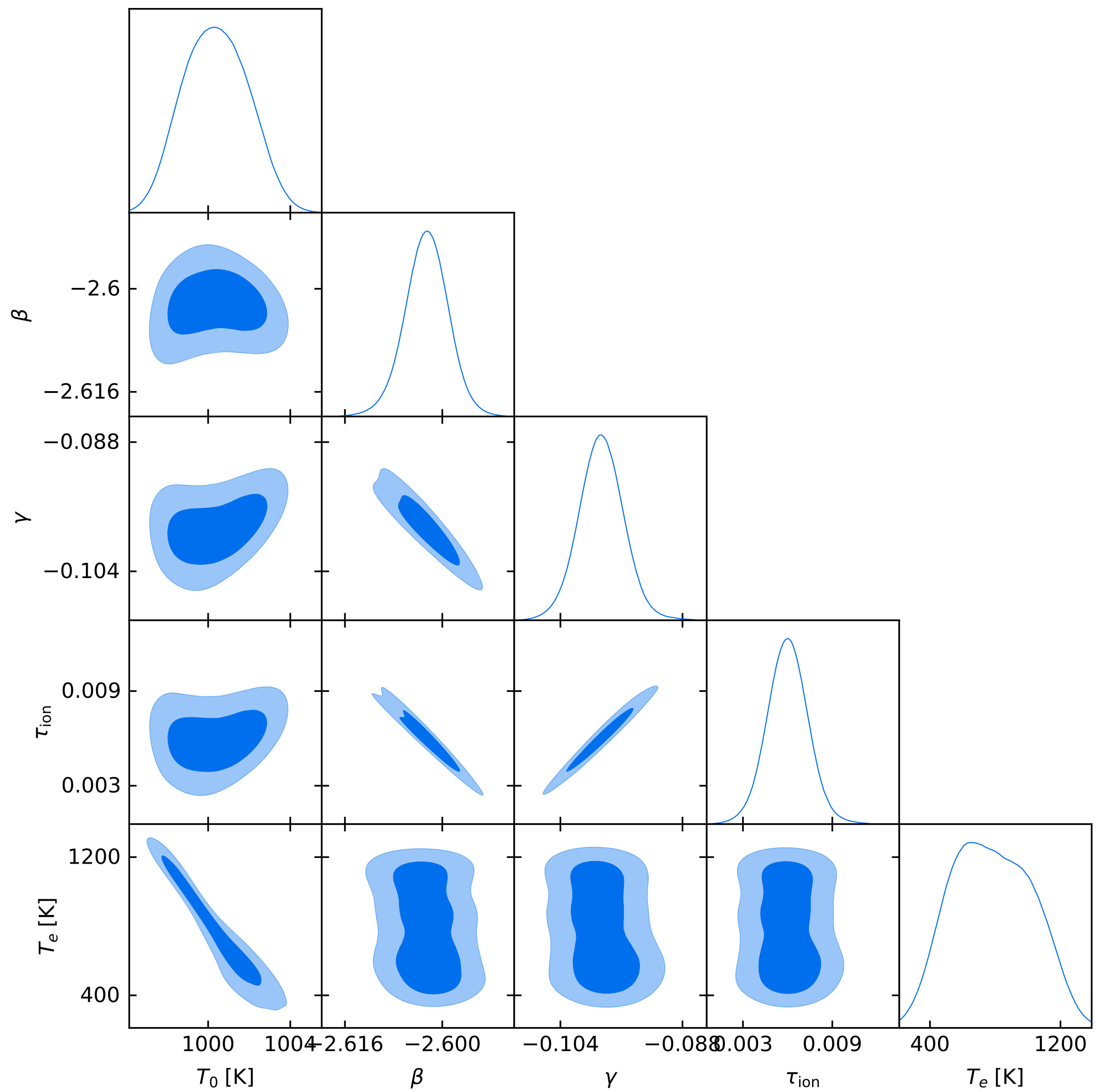

Figure 2: CASE 2. colors correspond to 68% and 95% confidence regions respectively.

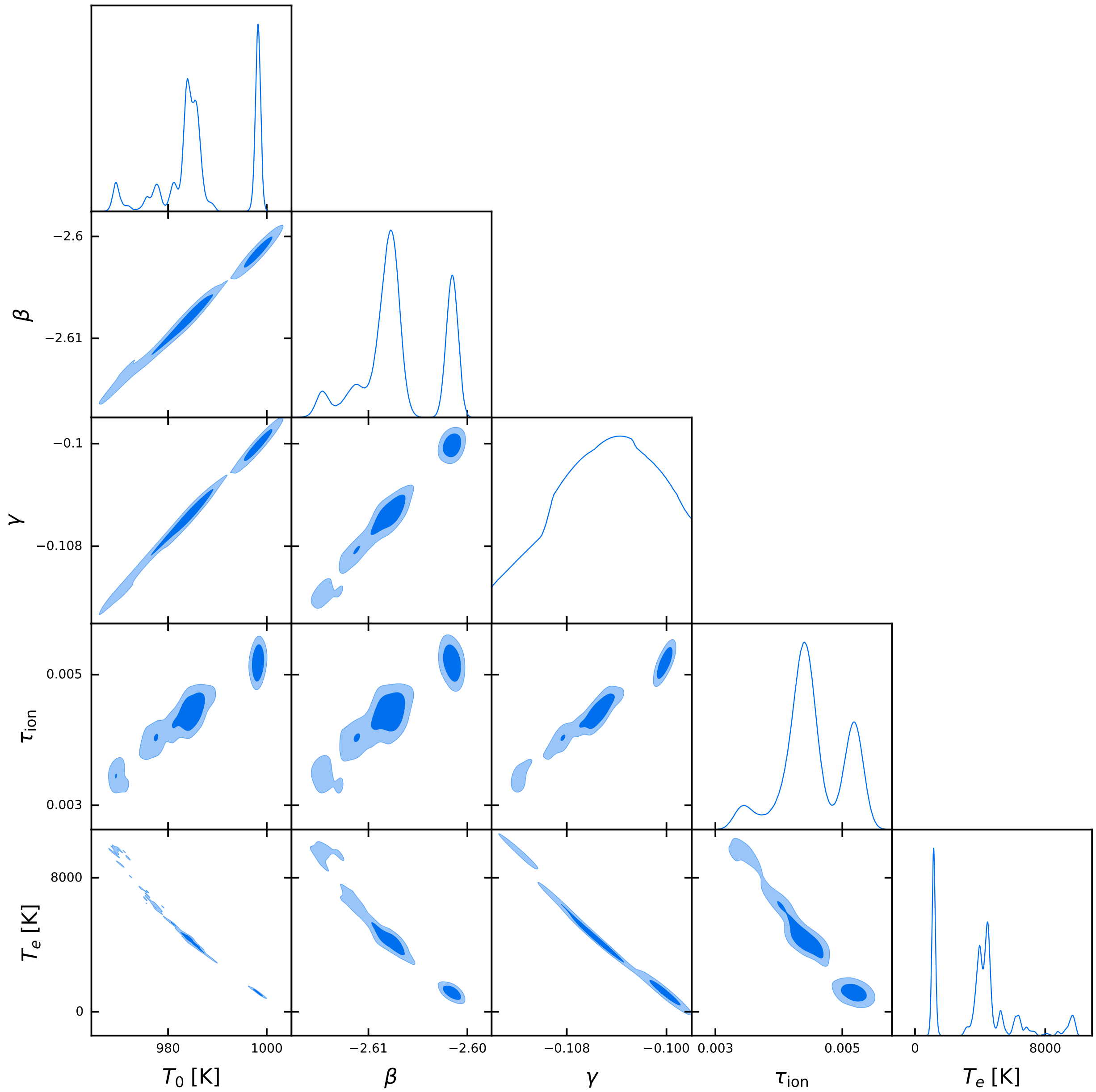

Figure 3: CASE 3. colors correspond to 68% and 95% confidence regions respectively.

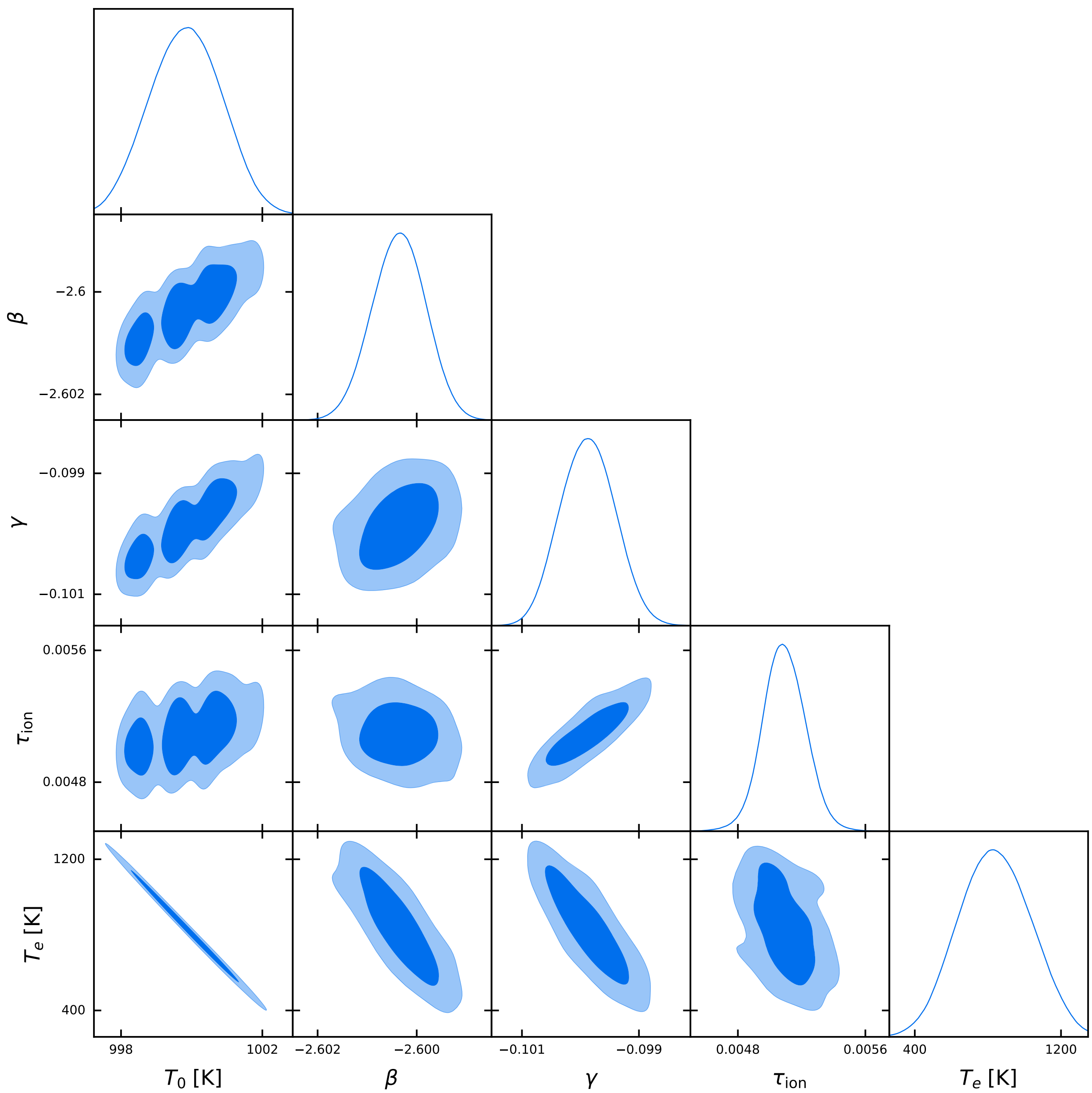

Figure 4: CASE 4. colors correspond to 68% and 95% confidence regions respectively.

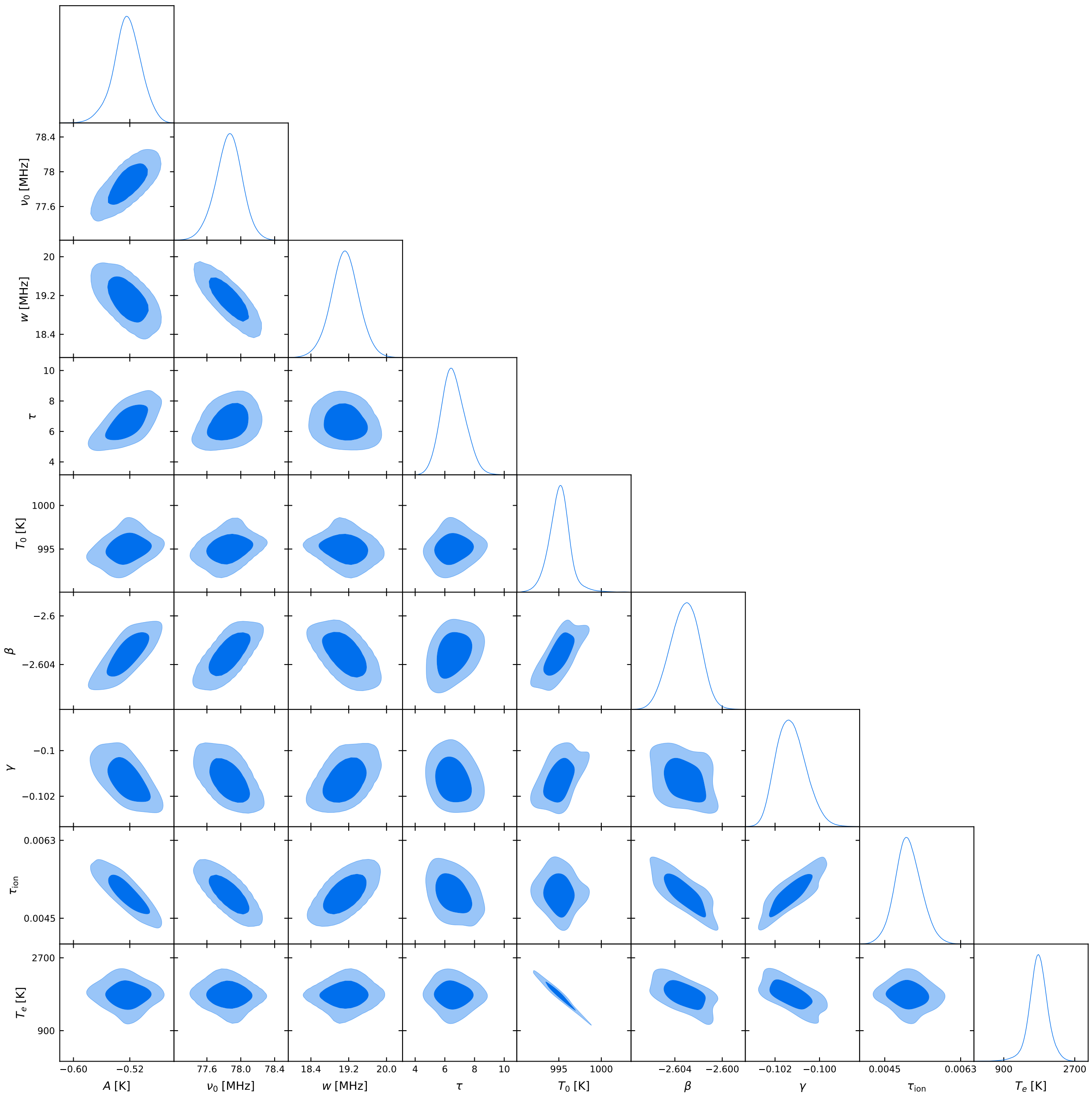

Figure 5: CASE 5. colors correspond to 68% and 95% confidence regions respectively.

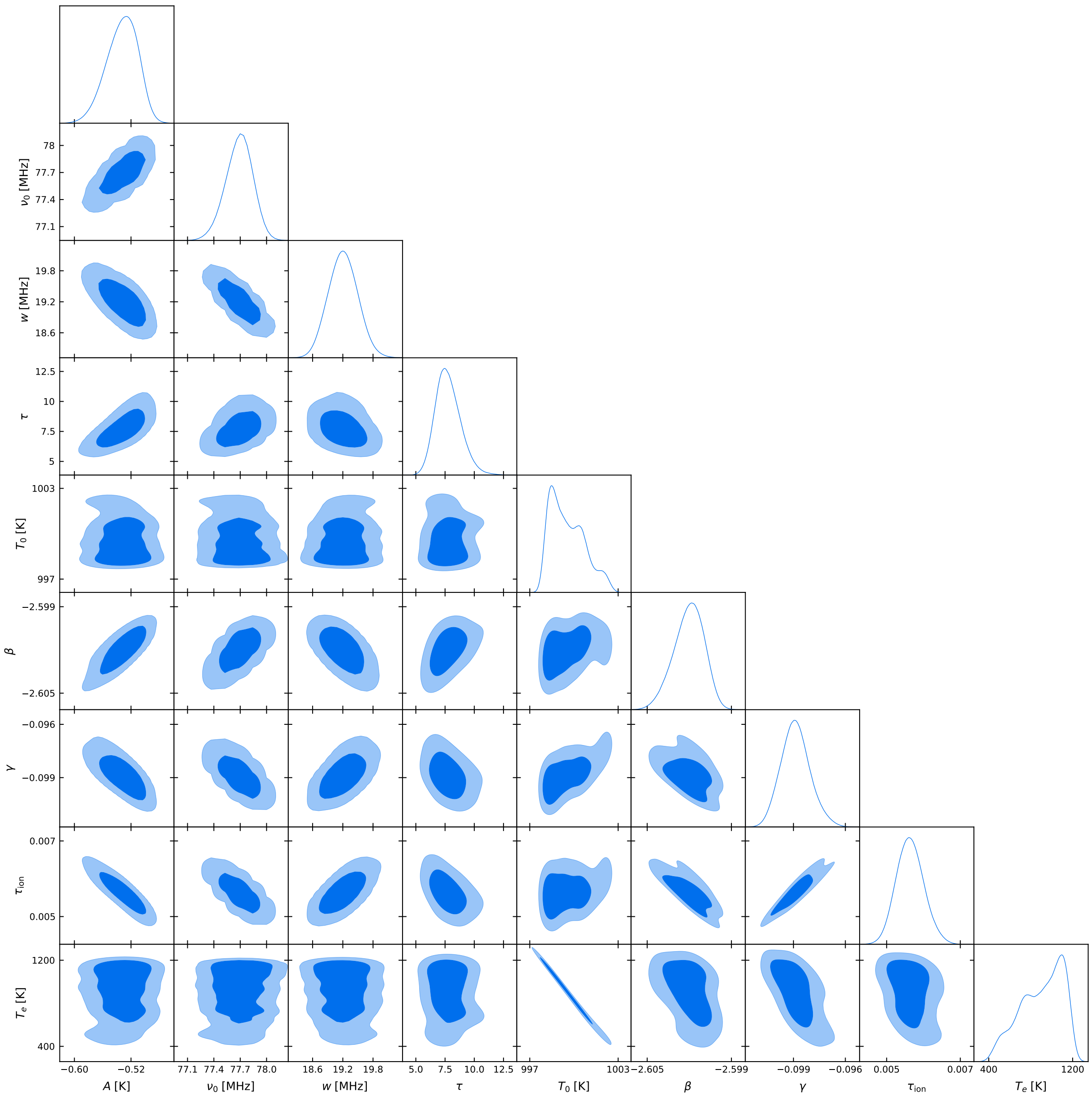

Figure 6: CASE 6. colors correspond to 68% and 95% confidence regions respectively.

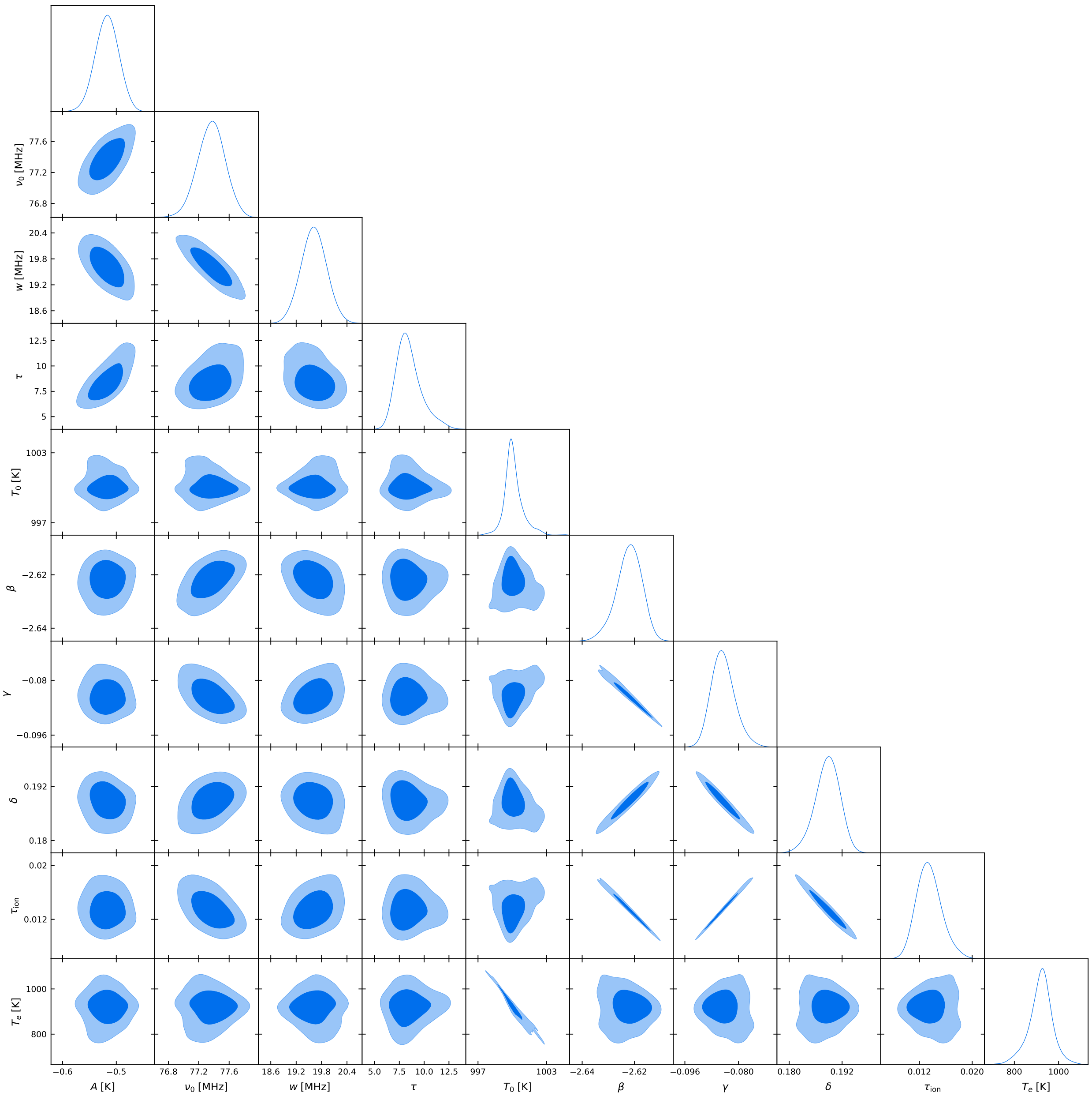

Figure 7: CASE 7. colors correspond to 68% and 95% confidence regions respectively.