LoCo Lab EDGES Memo 187
Computation of Tropospheric/Ionospheric Refraction in Single-Antenna Measurements

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1 Description

Here we show an expression to compute the antenna temperature including the effects of tropospheric and ionospheric refraction on single-antenna observations. To include these effects, the strategy here is to compute the convolution with a stretched version of the original antenna beam. It is necessary to also normalize the convolution by the stretched beam solid angle.

2 Generic Notation

In the case without refraction, the antenna temperature at each LST is given by

$$\hat{T}_{\mathrm{A}} = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\frac{\pi}{2}} G(\theta,\phi)\cdot T_{\mathrm{sky}}(\theta,\phi)\cdot \sin\theta d\theta d\phi \tag{1}$$

where G is the beam gain and T_{sky} is the sky brightness temperature. To include refraction, three aspects have to be incorporated into the above expression: (1) The zenith angle of every beam gain point has to shift from θ to $\theta+\delta\theta(\theta)$, where $\delta\theta(\theta)$ is the refraction angle, which itself is a function of θ. (2) The integration limit for θ has to change from $\pi/2$ to $\pi/2+\delta\theta(\pi/2)$. (3) A factor has to be included to normalize the convolution by the now larger beam solid angle. The following equation incorporates these three aspects:

$$\begin{aligned}\hat{T}_{\mathrm{A}} = &\left[\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\frac{\pi}{2}+\delta\theta\left(\frac{\pi}{2}\right)} G(\theta-\delta\theta(\theta),\phi)\cdot T_{\mathrm{sky}}(\theta,\phi)\cdot \sin\theta d\theta d\phi\right] \\ &\times\left[\frac{\int_0^{2\pi}\int_0^{\frac{\pi}{2}} G(\theta,\phi)\cdot\sin\theta d\theta d\phi}{\int_0^{2\pi}\int_0^{\frac{\pi}{2}+\delta\theta\left(\frac{\pi}{2}\right)} G(\theta-\delta\theta(\theta),\phi)\cdot \sin\theta d\theta d\phi}\right].\end{aligned} \tag{2}$$

3 Healpix Notation

The same equations could be written in the following way to represent operations on arrays, in particular where the solid angle of each sky temperature pixel is the same, such as when working in the Healpix format. The case without refraction is:

$$\hat{T}_{\mathrm{A}} = \frac{1}{N_{\mathrm{pix}}}\sum_{\phi=0}^{2\pi}\sum_{\theta=0}^{\frac{\pi}{2}} G\cdot T_{\mathrm{sky}}, \tag{3}$$

while the case with refraction is:

$$\hat{T}_{\mathrm{A}} = \left[\frac{1}{N_{\mathrm{pix}}} \sum_{\phi=0}^{2\pi} \sum_{\theta=0}^{\frac{\pi}{2}+\delta\theta\left(\frac{\pi}{2}\right)} G^{*} \cdot T_{\mathrm{sky}} \right] \times \left[\frac{\sum_{\phi=0}^{2\pi} \sum_{\theta=0}^{\frac{\pi}{2}} G}{\sum_{\phi=0}^{2\pi} \sum_{\theta=0}^{\frac{\pi}{2}+\delta\theta\left(\frac{\pi}{2}\right)} G^{*}} \right] \tag{4}$$

where N_{pix} is the number of pixels across the full sphere, and G^{*} is the stretched beam.

4 Examples

In the figure below we show the effect of tropospheric/ionospheric refraction on simulated observations of the Guzman et al. and LW maps at 45 and 150 MHz respectively.

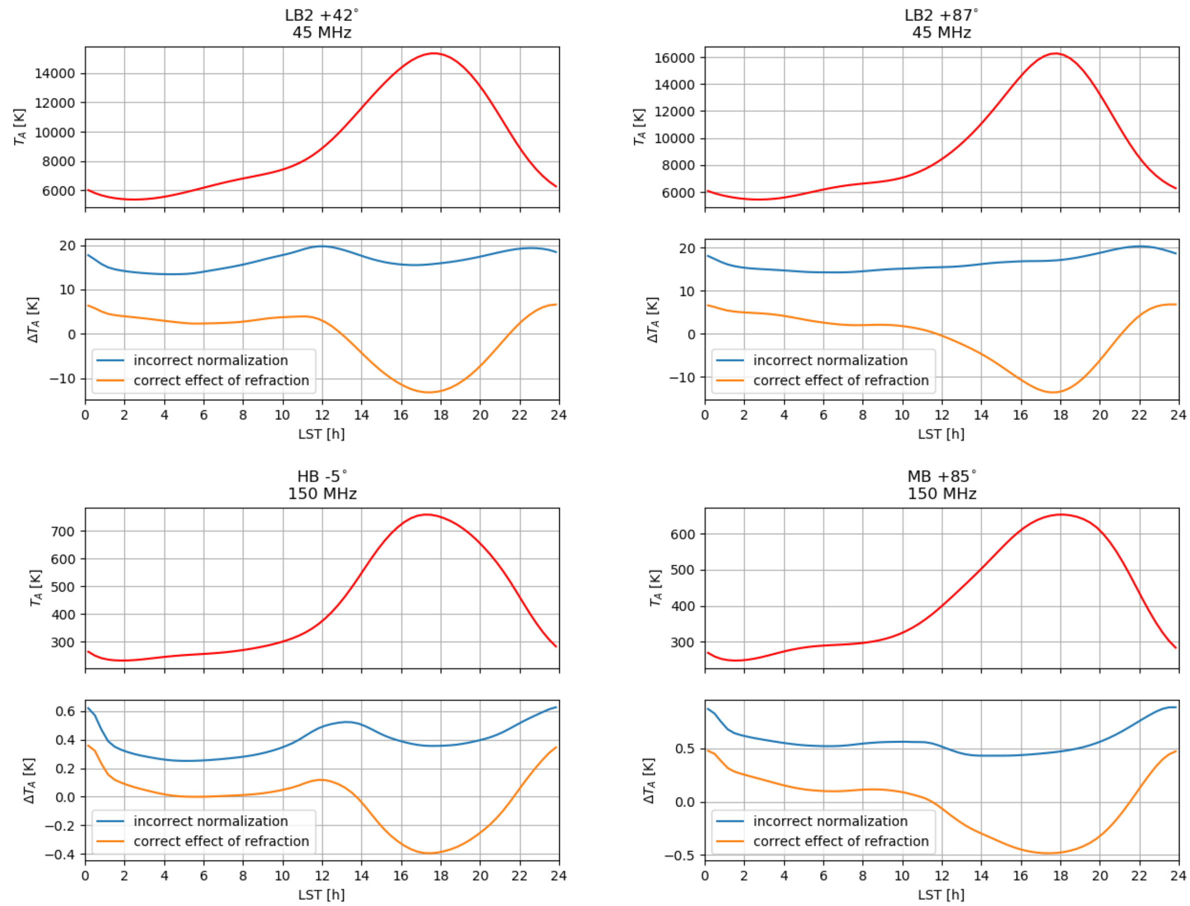

Figure 1: Simulated observations of Guzman et al. and LW maps, at 45 and 150 MHz respectively, with Low-Band 2 (LB2 +42° and +87°), High-Band (HB −5°), and Mid-Band (MB +85°). The red lines are the simulations without refraction. The orange lines correspond to the effect of refraction (i.e., simulation with refraction minus without refraction). In particular, during Galactic transit refraction makes the antenna temperature be lower than without refraction. The blue lines show the effect of computing refraction without normalizing by the larger solid angle (i.e., not including the second term in Equations 2 and 4).