Effect of Loss on VNA Calibration Standards

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August 7, 2013

Description

These slides have the purpose of summarizing the details of the modeling of calibration standards by Agilent, as presented in their Application Note 1287-11.

Using this recipe, the reflection coefficient of the standards in the Agilent calibration kit model 85033E is computed. This expectation is compared to measurements using the Agilent E5072A and R&S ZVL3 VNAs.

Reflection Coefficient of Lossy Calibration Standards

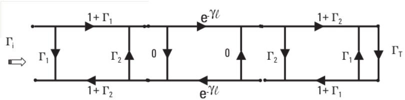

Figure: (1): Block diagram of the standards (Figure 4 in the Agilent Application Note).

According to equation 1.4 of the Agilent Application Note 1287-11, depicted in figure 1, the reflection coefficient of a termination with intrinsic reflection coefficient Γ_T at the end of a lossy transmission line is

$$\Gamma_i = \frac{\Gamma_1\left(1 - e^{-2\gamma\ell} - \Gamma_1\Gamma_T\right) + e^{-2\gamma\ell}\Gamma_T}{1 - \Gamma_1\left[e^{-2\gamma\ell}\Gamma_1 + \Gamma_T\left(1 - e^{-2\gamma\ell}\right)\right]} \qquad (1)$$

where

$$\Gamma_1 = \frac{Z_c - Z_r}{Z_c + Z_r}, \qquad \Gamma_2 = -\Gamma_1,$$

$$\Gamma_T = \frac{Z_T - Z_r}{Z_T + Z_r},$$

ℓ;	length of transmission line
$\gamma = \alpha + j\beta$;	propagation constant
Z_c;	characteristic impedance of transmission line
Z_r;	reference or system impedance (50 Ω)
Z_T;	impedance of the termination.

The transmission line is characterized by its impedance, Z_c (and therefore Γ_1), and its propagation constant, γ.

Both quantities can be expressed in terms of the *offset impedance*, *offset delay*, and *offset loss*.

Impedance of Terminations

Instead of assuming that the standards have ideal behavior at the end of the transmission lines (Γ=1 for the *open*, -1 for the *short*, and 0 for the *match*), more realistic models have to be used:

- Open (only termination, no transmission line):

$$\begin{aligned} C_{open} &= C_o + C_1 f + C_2 f^2 + C_3 f^3, \\ Z_{open} &= \frac{-j}{2\pi f \cdot C_{open}}, \\ \Gamma_{open} &= \frac{Z_{open} - Z_r}{Z_{open} + Z_r}. \end{aligned} \tag{2}$$

- Short (only termination, no transmission line):

$$\begin{aligned} L_{short} &= L_o + L_1 f + L_2 f^2 + L_3 f^3, \\ Z_{short} &= j \cdot 2\pi f \cdot L_{short}, \\ \Gamma_{short} &= \frac{Z_{short} - Z_r}{Z_{short} + Z_r}. \end{aligned} \tag{3}$$

- Match (only termination, no transmission line):

$$\begin{aligned} Z_{match} &\neq 50\ [\Omega], \\ \Gamma_{match} &= \frac{Z_{match} - Z_r}{Z_{match} + Z_r}. \end{aligned} \tag{4}$$

Lossy Transmission Lines

The characteristic impedance and propagation constant for a general (lossy) transmission line are

$$Z_c = \sqrt{\frac{R + j\omega L}{G + j\omega C}}, \quad \gamma = j\omega\sqrt{LC}\sqrt{1 - j\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right) - \frac{RG}{\omega^2 LC}}. \tag{5}$$

Where R, G, C, and L are the distributed parameters. Due to the skin effect, the inductance can be written as

$$L = L_i + L_o = \frac{R}{\omega} + L_o. \tag{6}$$

In the approximation $G = 0$, the equations become

$$\begin{aligned} Z_c &\approx \sqrt{\frac{R + j\omega\left(L_o + \frac{R}{\omega}\right)}{j\omega C}} = \sqrt{\frac{L_o}{C}}\sqrt{1 + \frac{R}{\omega L_o}(1-j)}, \\ \gamma &\approx j\omega\sqrt{\left(L_o + \frac{R}{\omega}\right)C}\sqrt{1 - \frac{jR}{\omega\left(L_o + \frac{R}{\omega}\right)}} = j\omega\sqrt{L_oC}\sqrt{1 + \frac{R}{\omega L_o}(1-j)}. \end{aligned} \tag{7}$$

Expanding there expressions to first order produces

$$\begin{aligned} Z_c &= \sqrt{\frac{L_o}{C}}\left[1 + (1-j)\left(\frac{R}{2\omega L_o}\right)\right], \\ \gamma &= j\omega\sqrt{L_oC}\left[1 + (1-j)\left(\frac{R}{2\omega L_o}\right)\right]. \end{aligned} \tag{8}$$

Lossy Transmission Lines

The parameters in the last two equations are related to the *offset impedance*, (one-way) *offset delay*, and (one-way) *offset loss* as

$$
\begin{aligned}
\text{offset } Z_o \quad [\Omega] &= \sqrt{\frac{L_o}{C}}, \qquad \text{(characteristic impedance without loss)} \\
\text{offset delay} \quad [\text{s}] &= \frac{\ell\sqrt{\epsilon_r}}{c} = \sqrt{L_o C}, \qquad (c\text{: speed of light, } C\text{: capacitance}) \\
\text{offset loss} \quad [\Omega/\text{s}] &= \frac{R\ell}{\sqrt{\frac{f}{10^9}} \cdot \text{offset delay}}.
\end{aligned}
\tag{9}
$$

Finally, the characteristic impedance and propagation constant can be written as [App. Note, eqn 2B.5]

$$
\begin{aligned}
Z_c &= \left[(\text{offset } Z_o) + \left(\frac{\text{offset loss}}{2\omega}\right)\sqrt{\frac{f}{10^9}}\right] - j\left[\left(\frac{\text{offset loss}}{2\omega}\right)\sqrt{\frac{f}{10^9}}\right], \\
\gamma\ell &= \left[\frac{(\text{offset loss})(\text{offset delay})}{2(\text{offset } Z_o)}\sqrt{\frac{f}{10^9}}\right] + j\left[\omega(\text{offset delay}) + \frac{(\text{offset loss})(\text{offset delay})}{2(\text{offset } Z_o)}\sqrt{\frac{f}{10^9}}\right].
\end{aligned}
\tag{10}
$$

Test with Calibration Kit Agilent 85033E

The reflection coefficient of the standards (terminations + lossy transmission lines) are computed for the 3.5-mm male *open*, *short*, and *match* of the Agilent 85033E calibration kit, using equation 1 and the results of equations 2, 3, 4, and 10.

Table 1 presents the parameters for the models of the standards.

Table: Parameters of the 3.5-mm male Agilent 85033E Standards

Parameter	Unit	OPEN	SHORT	MATCH
C_0	$\times 10^{-15}$ [F]	+49.43		
C_1	$\times 10^{-27}$ [F/Hz]	-310.1		
C_2	$\times 10^{-36}$ [F/Hz^2]	+23.17		
C_3	$\times 10^{-45}$ [F/Hz^3]	-0.1597		
L_0	$\times 10^{-12}$ [H]		+2.077	
L_1	$\times 10^{-24}$ [H/Hz]		-108.5	
L_2	$\times 10^{-33}$ [H/Hz^2]		+2.171	
L_3	$\times 10^{-42}$ [H/Hz^3]		-0.01	
termination resistance	[Ω]			50
offset Z_0	[Ω]	50	50	50
offset delay	[ps]	29.242	31.785	0
offset loss	[GΩ/s]	2.2	2.36	2.3

The following figure compares the calculated response of the standards, with measurements using the Agilent E5072A and the R&S ZVL3, after calibrating both instruments with the same kit, Agilent 85033E.

Test with Calibration Kit Agilent 85033E

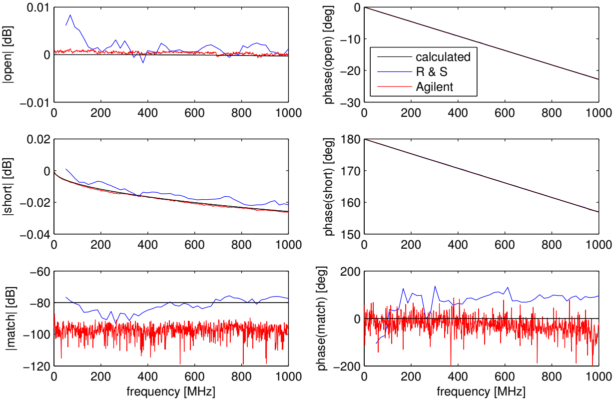

Figure: (2): Calculated and measured reflection coefficient of the male 3.5-mm standards of the Agilent 85033E calibration kit. The measurements were performed with the Agilent E5072A and R&S ZVL3 VNAs after calibrating both instruments with the same kit (with their definition files loaded). The resolution of the measurement with the R&S is lower to reduce its noise. No corrections of any kind were applied to the measurements. A value of 50.01 Ω was used for the termination resistance of the *match* in the calculation (black line), in order to have a finite value of reflection coefficient in dB scale. The most relevant points of comparison are the magnitude of the *open* and *short* (upper and middle left plots). The main conclusion is that the *offset loss* affects significantly the reflection coefficient of the *short*, but not so that of the *open*. For the *open* it goes from -1×10^{-11} dB at 1 MHz to -3×10^{-4} dB at 1 GHz. This is very flat compared to the behavior observed for the *short*. In this regard, the measurements from both VNAs agree with the calculations. Of course, the agreement is much better with the Agilent VNA.