Studying the Forward-Reverse Method Through Simulations

Raul Monsalve

SESE, Arizona State University

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Description

This report describes the ability of the Forward-Reverse Method (FRM) to estimate parameters of the VNA calibration standards from measurements at a single frequency, f = 1 GHz. In this method, the *open*, *short*, and *load* standards are measured through a test network with S-parameters *St*, connected in the forward and reverse directions. A previous study recommends the following values for the network: $|St_{11}| = 0.1$, $|St_{22}| = 0.9$, and $Phase(St_{12}St_{21}) = 180^{\circ}$ (which is equivalent to $Phase(St_{12}) = 90^{\circ}$). In addition, a network *Se* is used to model imperfections of the VNA calibration.

Simulated measurements are generated, in which:

1. The response of the standards is computed using their models and assigning a set of values to the parameters.
2. The standards are measured at the measurement plane, which is separated from 50-Ω by the *Se* network.
3. The standards are measured at the end of network *St* connected in forward mode.
4. The standards are measured at the end of network *St* connected in reverse mode.
5. Uncorrelated noise is assigned to the real and imaginary parts of each measurement.

Parameter values are estimated from these simulations by minimizing the figure of merit (FoM):

$$FoM = |St_{11f} - St_{11r}| + |St_{12f}St_{21f} - St_{12r}St_{21r}| + |St_{22f} - St_{22r}|, \qquad (1)$$

where *f* and *r* represent the forward and reverse directions. The *St* network is computed from the synthetic measurement using the de-embedding method, after also de-embedding *Se*. The best set of parameters would allow to calculate *Se* and *St* such that FoM is minimized.

The variance and covariance of the estimated parameters are quantified by simulating each of the considered scenarios 500 times (or repetitions).

Description

The simulations allow to evaluate different scenarios:

- Different numbers of free parameters.
- Different fiducial values for the parameters.
- Different values for Se.
- Different values for St.
- Different values for the measurement noise.

This report presents simulations primarily divided into two categories:

- One free parameter: the delay of the 50-Ω LOAD.
- Three free parameters: the loss of the SHORT, the loss of the LOAD, and the delay of the LOAD.

In both cases, the nominal simulation settings are:

Simulation Parameter	Value	Comments
Number of repetitions	500	Sufficient repetitions for sampling parameter space in general
Frequency	1 GHz	The figure of merit is more sensitive as frequency increases
SHORT loss	2.36 GΩ/s	
LOAD loss	2.30 GΩ/s	
LOAD delay	30 ps	
Other parameters of standards	nominal	
Se_{11}	$0, 0^\circ$	Perfect VNA calibration
$Se_{12}Se_{21}$	$1, 0^\circ$	Perfect VNA calibration
Se_{22}	$0, 0^\circ$	Perfect VNA calibration
St_{11}	$0.1, 45^\circ$	close to optimal test network
$St_{12}St_{21}$	$0.8, 180^\circ$	Close to optimal test network
St_{22}	$0.9, 45^\circ$	Close to optimal test network
Measurement noise	1e-5	Approximate 1σ noise level for all loads in real and imaginary domains

The following figures present the results of simulations with nominal settings, compared to cases with other realistic settings.

One Free Parameter: LOAD delay

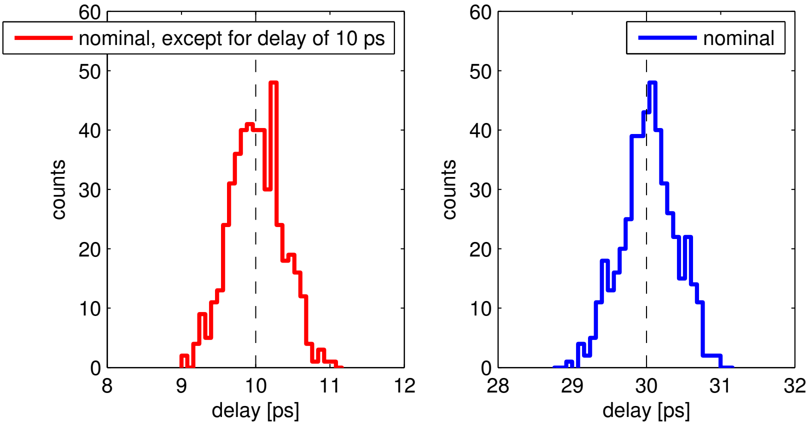

Figure : (1): the uncertainty is similar for all true values of delay.

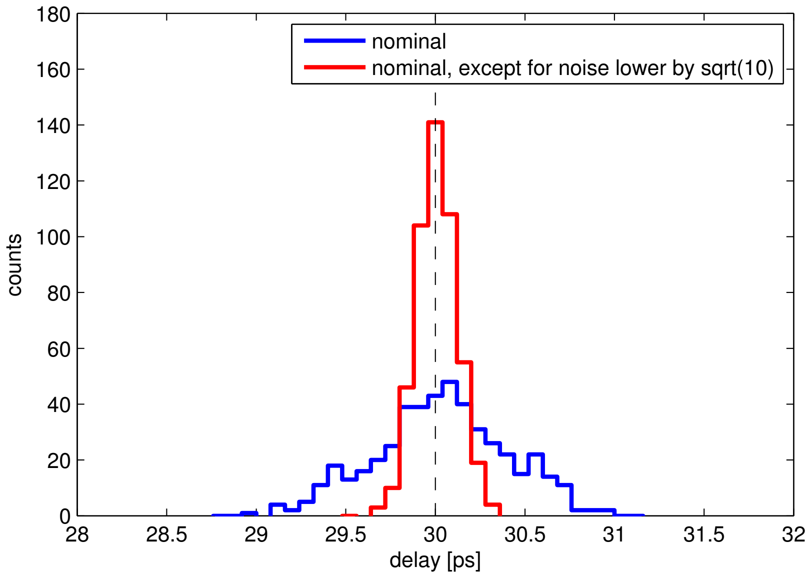

Figure : (2): when averaging 10 measurements, the recovered precision improves significantly.

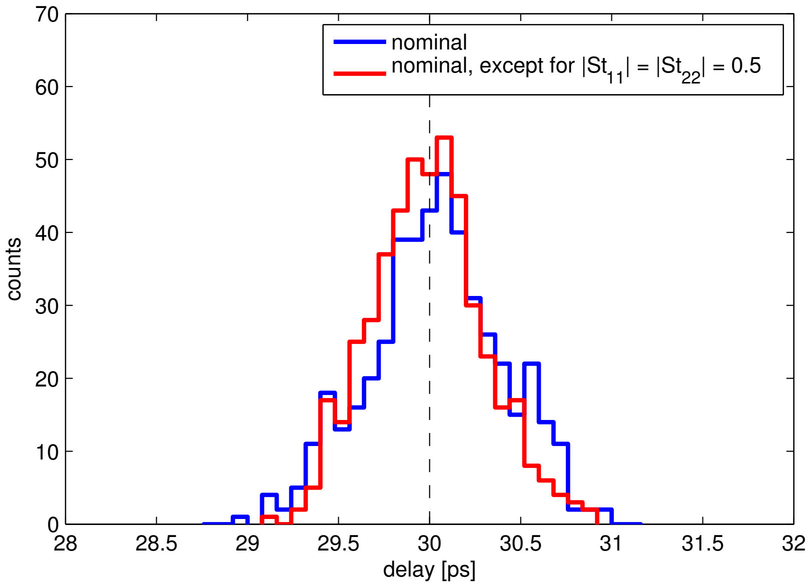

Figure : (3): the performance of the parameter estimation is not very sensitive to $|St_{11}|$ and $|St_{22}|$.

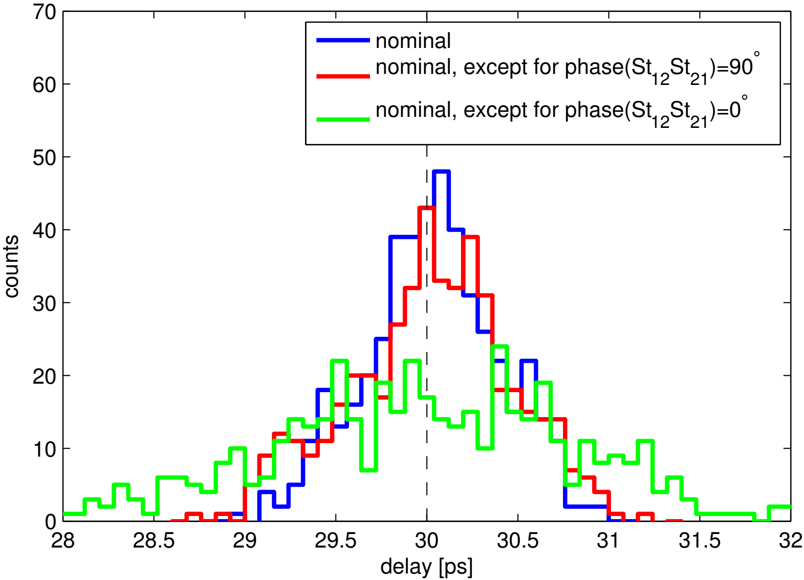

Figure : (4): the performance of the parameter estimation is sensitive to the phase of $St_{12}St_{21}$. The best value is 180°.

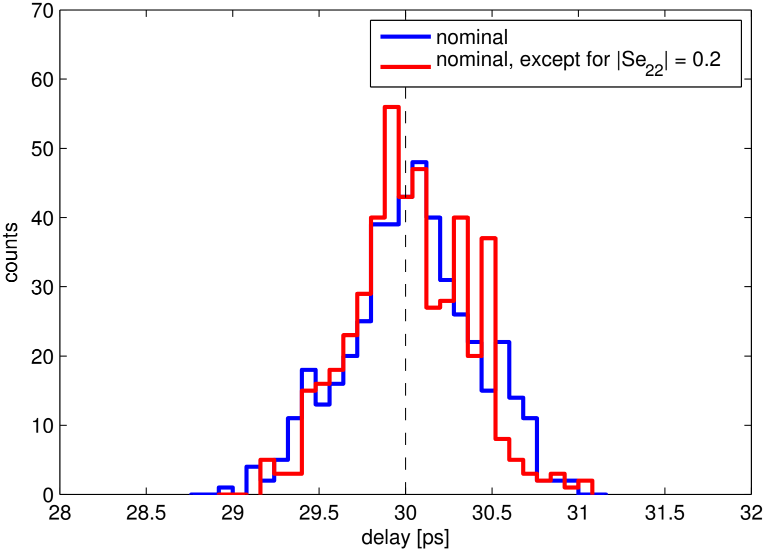

Figure : (5): the performance of the parameter estimation is not very sensitive to $|Se_{11}|$ and $|Se_{22}|$. Here, only the effect of a change in $|Se_{22}|$ in shown.

Three Free Parameters:

- SHORT loss
- LOAD loss
- LOAD delay

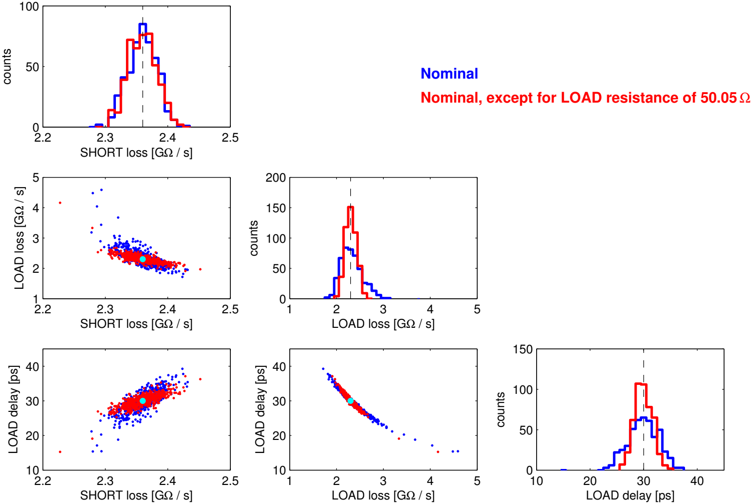

Figure : (6): obviously, increasing the free parameters to three increases the uncertainty in the LOAD delay. In this scenario, the loss of the SHORT is the parameter that can be estimated with the best precision. If the resistance of the LOAD departs from 50 Ω, the uncertainty of its delay and loss decreases (RED is better than BLUE).

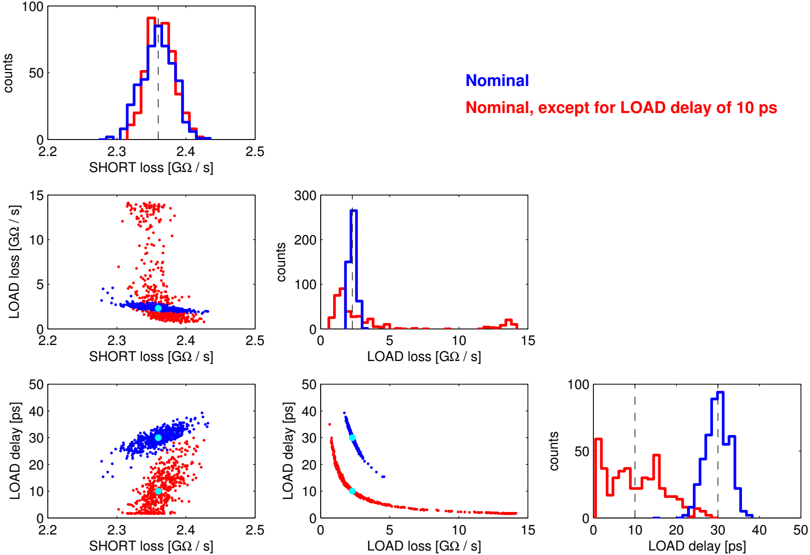

Figure : (7): under otherwise equal conditions, when the LOAD delay is closer to zero (10 ps vs 30 ps), the uncertainty of the LOAD delay and loss increases (there is a heavy $\sim 1/x$ correlation between them). This change has no impact on the SHORT loss.

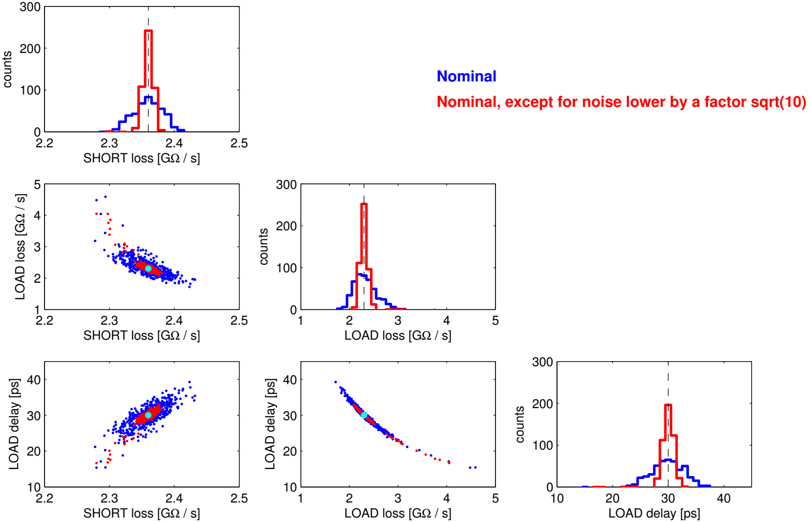

Figure : (8): when typical measurements are averaged 10 times, the precision improves significantly. This case corresponds to a LOAD delay of 30 ps.

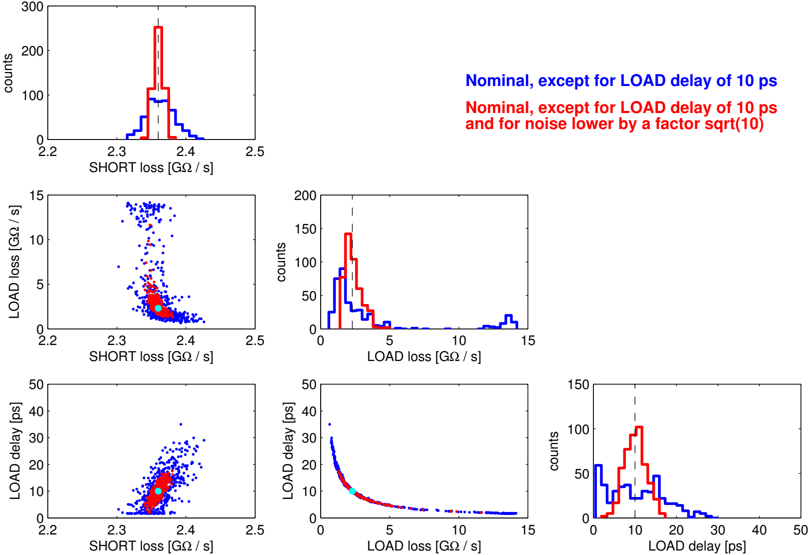

Figure : (9): when the LOAD delay is close to zero, the averaging of 10 measurements also produces an important improvement in the uncertainty of the LOAD delay and loss, but it is clearly worse than for higher values of delay.

Conclusion

- When the FRM is used to estimate only the LOAD delay, it could achieve a precision $\sim$ 1 ps or better even when the delay is close to zero.
- When three parameters are estimated, the LOAD loss and delay could achieve a precision $\sim$ 0.2 GΩ/s and 1 ps only if the LOAD delay is not close to zero, even with reduced noise.
- If the LOAD delay is close to zero, the precision of the recovered loss and delay degrades. For a delay of 10 ps and reduced noise, the precisions are $\sim$ 2 GΩ/s and $\sim$ 5 ps respectively.
- The precision of the SHORT loss is always better than 0.1 GΩ/s when three parameters are estimated.
- All of the above is valid if there are no systematic effects or they are at most comparable to the noise.