Rejection of Tanh Models for the Global 21-cm Signal from Simulated Data *(ongoing work)*

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Description

Here we show preliminary constraints on the EoR transition, which is modeled with the Tanh expression, i.e.:

$$T_{21}(z) = A_{21}\,\frac{1}{2}\,\left[\tanh\left(\frac{z-z_r}{\Delta z}\right)+1\right]\sqrt{\frac{1+z}{10}}. \tag{1}$$

The foregrounds are modeled with the EDGES polynomial,

$$T_{\rm fg}(\nu) = \sum_{i=0}^{N_{\rm fg}-1} a_i \nu^{-2.5+i}. \tag{2}$$

We estimate the parameters of our linear model, $\boldsymbol{\lambda} = [A_{21}, \{a_i\}]$, for each combination of z_r and Δz. We employ maximum likelihood assuming Gaussian errors and write

$$\hat{\boldsymbol{\lambda}} = \left(M^T W M\right)^{-1} M^T W d, \tag{3}$$

where $\hat{\boldsymbol{\lambda}}$ is the parameter vector estimate, M is the design matrix, with columns that correspond to the 21 cm function (equation 1 with $A_{21} = 1$) and the foreground polynomial terms (terms of equation 2 with $a_i = 1$), and W is a diagonal matrix of weights, which we set equal to the number of raw data points that goes into each frequency channel. The covariance matrix of the parameters is computed as

$$\hat{\Sigma}_{\boldsymbol{\lambda}} = s^2\left(M^T W M\right)^{-1}. \tag{4}$$

Here, s^2 is the unbiased weighted mean squared error (equivalent to the reduced chi-squared statistics), defined as

$$s^2 = \frac{r^T W r}{N_\nu - N_\lambda - 1}, \tag{5}$$

where r is the difference between the data and the best fit model, N_ν is the number of frequency points, and N_λ is the number of parameters.

Description

The figures on the next pages present the following calculations or results:

1. Simulated upper limits for constraints, from error bars alone.
2. Published forecasts, from error bars alone.
3. Simulated dependence of constraints on data realization.
4. Simulated reduction of constraints due to systematics.
5. Simulated reduction of constraints due to EoR signal, using 5 foreground terms.
6. Simulated reduction of constraints due to EoR signal, using 14 foreground terms.

Simulated upper limits for constraints, from error bars alone

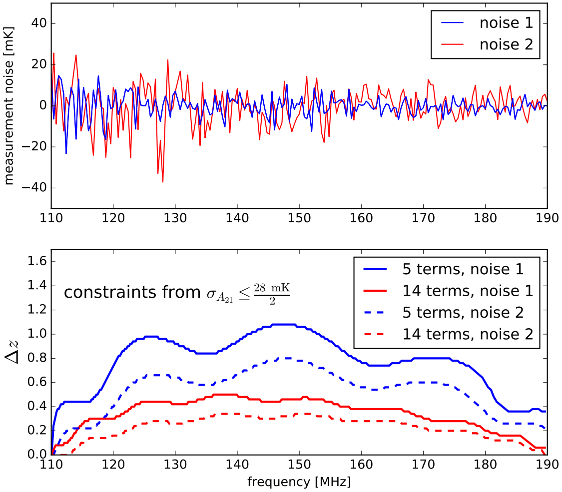

Figure: (1): TOP: Two realizations of noise with different standard deviation, added to a power law simulating the foregrounds. BOTTOM: Constraints based only on the size of the error bar of $\hat{A}_{21}$, for the two noise levels, and two foreground models, which have 5 and 14 terms respectively.

Published forecasts, from error bars alone

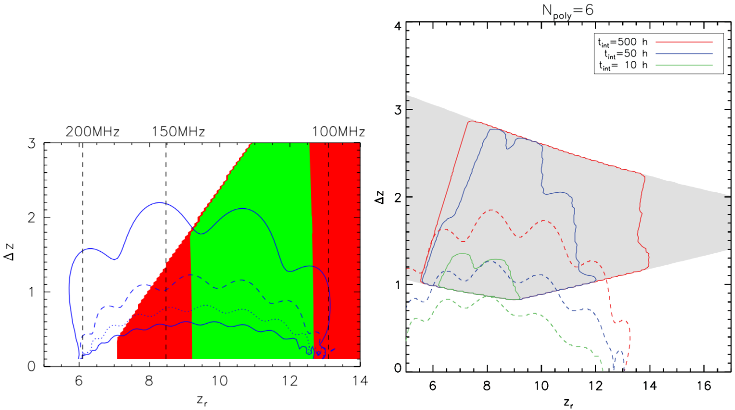

Figure: (2): Published forecasts based only on the size of the error bar of $\hat{A}_{21}$, which is equivalent to a Fisher matrix analysis. LEFT: From Pritchard and Loeb (2010). RIGHT: From Morandi and Barkana (2012).

Simulated dependence of constraints on data realization

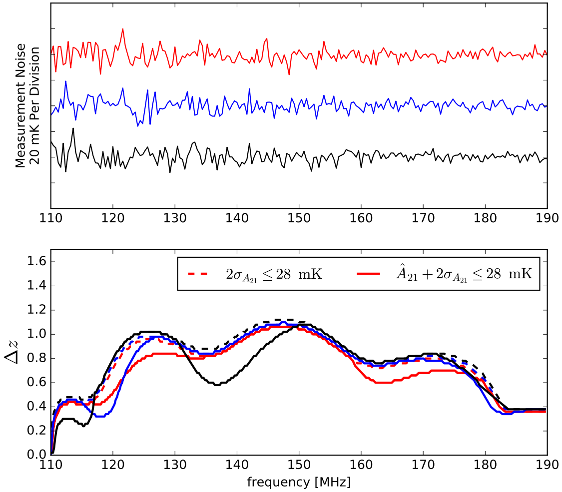

Figure: (3): TOP: Three realizations of noise with same standard deviation. BOTTOM: Constraints from three noise realizations for a 5-term foreground model, using (DASHED) error bars alone, and (SOLID) error bars and best-fit estimates. In the second case (SOLID), the different noise produces significant differences in the constraints.

Simulated reduction of constraints due to systematics

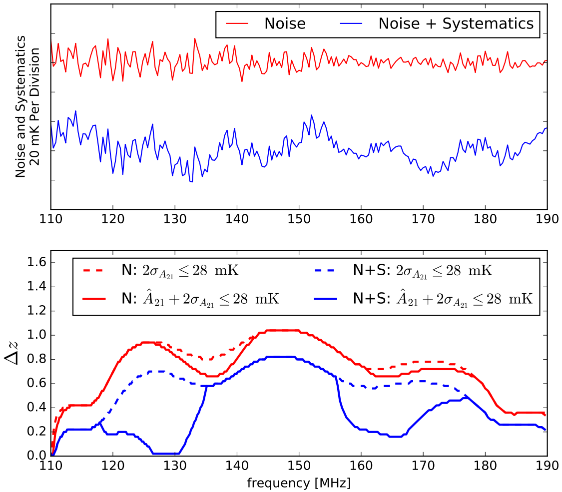

Figure: (4): TOP: Noise (RED) and Noise + Systematics (BLUE) added to the simulated foreground power law. BOTTOM: Constraints for 5-term foreground model and the noise and systematics from top panel. Dashed lines correspond to constraints from error bars alone, and solid lines are constraints from error bars and best-fit estimates. Clearly, the presence of systematics impacts the constraints.

Simulated reduction of constraints due to EoR signal

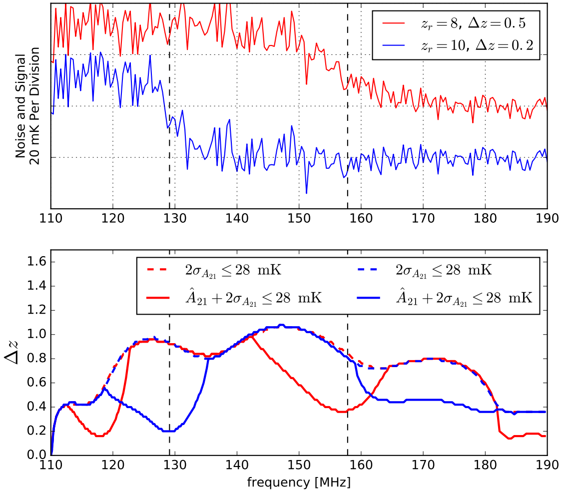

Figure: (5): TOP: Noise + Tanh EoR signal added to simulated foreground power law. BOTTOM: Constraints for 5-term foreground model. The rejections correctly avoid regions that include the injected EoR models. The dashed vertical black lines represent the reference frequencies (equivalently, z_r) of the two EoR models.

Simulated reduction of constraints due to EoR signal

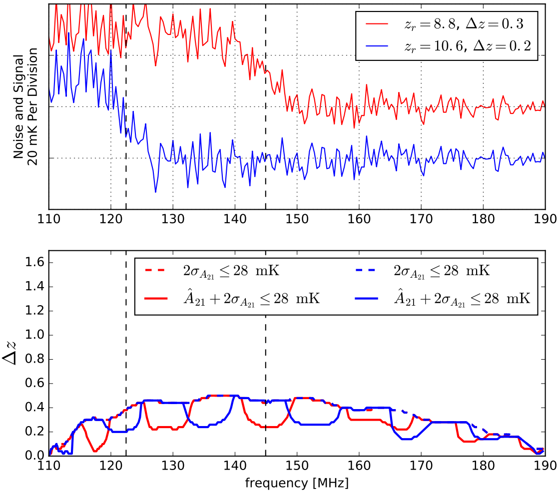

Figure: (6): Similar to Figure 5, but for a 14-term foreground model. TOP: Two realizations of noise and EoR models. BOTTOM: The rejections avoid the regions that include the injected EoR models.