21-cm Parameter Estimation using Markov Chain Monte Carlo

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Summary

In this report we present the use of Markov Chain Monte Carlo (MCMC) for estimating phenomenological parameters of the global redshifted 21-cm signal. We use this technique with a real integrated measurement of the sky-average brightness temperature spectrum conducted with EDGES Low-Band 1.

A non-exhaustive list of previous work that employs MCMC in the context of simulations includes Harker et al. [2012, 2016]; Mirocha et al. [2015], while it has only been applied once to the analysis of real data, in Bernardi et al. [2016], to estimate a Gaussian phenomenological model for the absorption trough from First Light.

Analysis Details

The EDGES average spectrum used here corresponds to Low-Band 1 with the extended ground plane, and the following:

- Days: 2016-260 to 2017-091
- LST coverage: 23.76 - 11.76 Hr
- Sun elevation: $< -10^\circ$ (nighttime)
- Moon elevation: $< 0^\circ$
- Receiver calibration: 2015-08
- Antenna reflection coefficient: 2017-93
- Beam correction: yes
- Beam file: azelq_blade9perf7low_g4w
- Panel AZ: -7° from North
- Sky model: Guzman-Haslam interpolation
- Ground loss correction: yes
- Ground loss correction value: 0.5% flat
- Balun loss correction: yes
- Averaging type: residuals

Analysis Details

For this exercise we fit a model to spectrum data in the range 61 − 99 MHz. The model is:

$$model = model_{fg} + model_{21} \tag{1}$$

where the foreground model is:

$$model_{fg} = \sum_{i=0}^{3} a_i \left(\frac{\nu}{\nu_n}\right)^{-2.5+i}, \tag{2}$$

with $\nu_n = 80$ MHz. The 21-cm model was introduced in memos 220 [1] and 226 [2],

$$model_{21} = -a_{21}\frac{K_1 K_2}{K_3}, \tag{3}$$

where:

$$K_1 = 1 - \exp\left(-\tau \exp\left(-4b\left[\frac{\nu-\nu_r}{\Delta\nu}\right]^2\right)\right) \tag{4}$$

$$b = -ln\left(-\frac{1}{\tau} ln\left(\frac{1}{2}\left[1+\exp(-\tau)\right]\right)\right) \tag{5}$$

$$K_2 = 1 + \chi\left(\frac{\nu-\nu_r}{\Delta\nu}\right) \tag{6}$$

$$K_3 = 1 - \exp\left(-\tau\right). \tag{7}$$

The fit parameters are $[a_0, a_1, a_2, a_3, a_{21}, \nu_r, \Delta\nu, \tau, \chi]$, where τ is a 'flatness' parameter and χ is a 'tilt' parameter. See memos 220 and 226 for details.

1 http://www.haystack.mit.edu/ast/arrays/Edges/EDGES_memos/220.pdf

2 http://www.haystack.mit.edu/ast/arrays/Edges/EDGES_memos/226.pdf

MCMC Configuration

The python MCMC implementation used is called emcee [3]. As with any implementation, some aspects that have to be defined include: (1) a model for the measurement noise, (2) the fit parameter priors, (3) the number of threads and the number of *walkers*, (4) the length of the MCMC chains, (5) the starting point of the chains, and (6) the burn in period in the exploration of the parameter space.

1. Measurement noise: The standard deviation of the noise is modeled, nominally, with a frequency independent value of 30 mK. We also use 60 mK.
2. Parameter priors: Given the high signal-to-noise anticipated, we only assign priors to the 21-cm parameters, not to the foreground parameters. However, they correspond to broad, uniform and uninformative priors, listed in the tables presented on the following slides.
3. Number of threads/walkers: We use 6 threads, chosen according to the processor cores available, and $2 \times (Npar + 1)$ walkers, which for 9 parameters is equal to 20.
4. Chain length: The only requisite here is to have enough useful samples to thoroughly explore parameter space. This is evaluated from the shape of the posterior distributions. We use a value of $10,000$ samples per walker.
5. Starting point in chains: The starting point for all parameters is drawn randomly from a uniform distribution with a width of 0.1, centered at realistic guesses.
6. Burn in period: The number of samples in the chains that are dismissed due to the chain evolving before converging to a stable region of parameter space is determined from the chains themselves. In our exercise this period is contained within the first 25% of the samples from each walker. Thus, we dismiss the first 30% of the samples from each walker.

[3] http://dan.iel.fm/emcee/current/

Nominal Exercise

The following table and figures describe our nominal configuration and results.

Table: (1) Inputs and results (±95% C.L.) for nominal exercise

	a_{21} [mK]	ν_r [MHz]	$\Delta\nu$ [MHz]	τ	χ
Initial guesses (+Uniform$[-0.05, +0.05]$)	500	78	20	3	0
Priors (Uniform)	$[0, 10000]$	$[65, 95]$	$[10, 30]$	$[0, 100]$	$[-1, 1]$
Estimates	678^{+43}_{-40}	$78.2^{+0.1}_{-0.1}$	$17.5^{+0.3}_{-0.3}$	$24.9^{+13.9}_{-7.7}$	$-0.32^{+0.08}_{-0.09}$

Figure 1 shows the full MCMC chains, with a total of $200,000$ samples, for each fit parameter. We dismiss the first 30% of the samples in each walker, considered to be in the burn in period.

Figure 2 shows the pairwise probability distributions, using the accepted chain samples.

Figure 3 shows the improvement in the residuals with the best fit 21-cm model. We also show all the realizations for the 21-cm model, corresponding to all the accepted MCMC samples.

Nominal Results

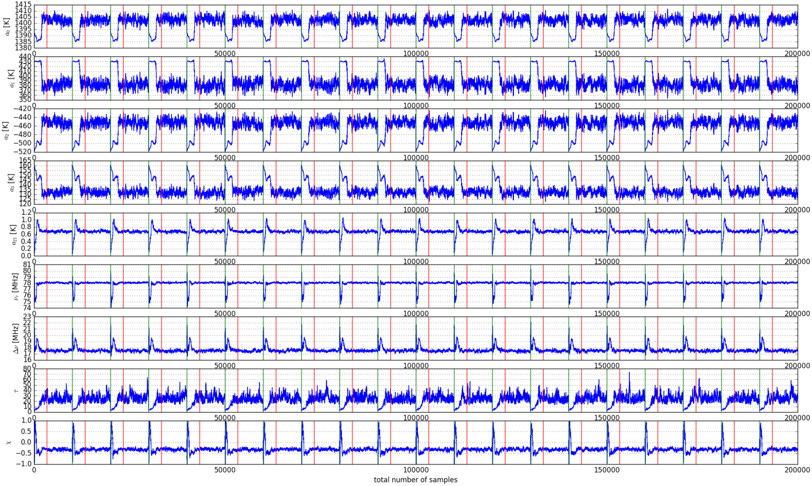

Figure: (1) Full MCMC chains for all the fit parameters. We used 20 walkers with 10, 000 samples each. Thus, the total number of samples is 200, 000. However, we dismiss the first 30% of the samples in each walker (between the green and red vertical lines) for considering them as part of the burn in period.

Nominal Results

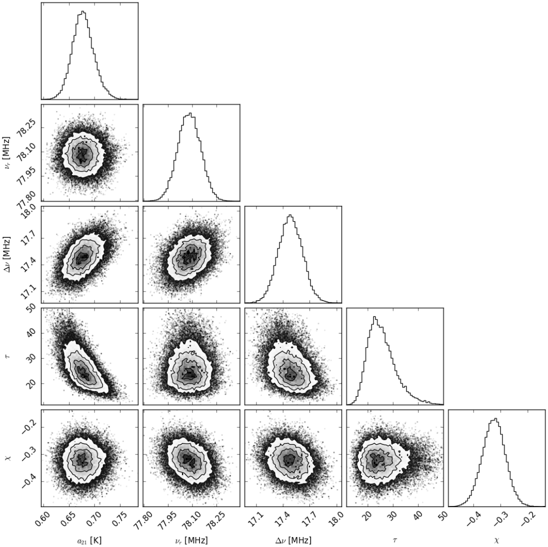

Figure: (2) 1D and pairwise marginalized posterior distributions for the 21-cm parameters. These distributions are sumarized in Table 1 in terms of the maximum likelihood values and 95% C.L.

Nominal Results

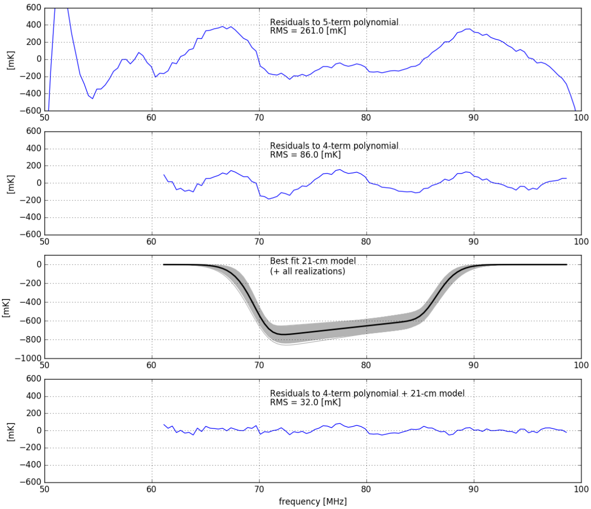

Figure: (3) *First panel*: residuals to 5-term polynomial fit over 50-100 MHz. *Second Panel*: residuals to 4-term polynomial fit over 61-99 MHz. *Third panel*: Best fit 21-cm model (thick black line), along with all model realizations for all accepted MCMC samples (gray). *Fourth panel*: residuals to full model, i.e., 4-term polynomial plus 21-cm model.

Additional Cases

Here, in Table 2 we show 2 additional results after changing the nominal MCMC configuration. The different cases represent:

- Case 1: change noise std from 30 mK to 60 mK flat.
- Case 2: change initial guesses from Uniform[-0.05, $+0.05$] centered at 500 mK, 78 MHz, 20 MHz, 3, 0, to Uniform[-5, $+5$] centered at 250 mK, 90 MHz, 12 MHz, 7, 0.9.

Table: (2) Results for different MCMC cases

	a_{21} [mK]	ν_r [MHz]	$\Delta\nu$ [MHz]	τ	χ
Nominal	678^{+43}_{-40}	$78.2^{+0.1}_{-0.1}$	$17.5^{+0.3}_{-0.3}$	$24.9^{+13.9}_{-7.7}$	$-0.32^{+0.08}_{-0.09}$
Case 1	664^{+86}_{-76}	$78.1^{+0.3}_{-0.3}$	$17.4^{+0.5}_{-0.5}$	$29.6^{+53.0}_{-15.6}$	$-0.33^{+0.17}_{-0.17}$
Case 2	679^{+47}_{-41}	$78.1^{+0.1}_{-0.1}$	$17.5^{+0.3}_{-0.3}$	$24.1^{+14.5}_{-8.1}$	$-0.32^{+0.09}_{-0.08}$

As expected, the higher noise increased the parameter uncertainties, and the change in initial guesses did not affect the estimates significantly due to the high signal-to-noise ratio of the spectrum.

Bibliography

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