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**Obtaining a Cold IGM through Modification of the  
 Residual Ionization Fraction Following Recombination**

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**1. Calculation of the Matter Temperature from  $z=1000$  to  $z\sim 15$**

The evolution of the baryonic matter (IGM) temperature before the formation of stars can be calculated in the linear regime by solving a small system of energy balance and radiative transfer equations. The first numerical code designed to solve the recombination physics at  $z>1000$  in order to model the CMB signal is RECFAST (see Seager et al. 2000 and related papers). Since it must utilize the same equations (and others) it can and has been used to calculate the matter temperature following recombination. More recent numerical codes are CosmoRec (Shaw & Chluba, 2011) and HyRec (Ali-Haimoud & Hirata, 2011). These codes are employed by Planck in their data analysis and are considered very accurate.

The essential equations governing the thermal evolution of matter following recombination are summarized in Seager et al. 2000 (ApJS, 128), their Section 2.5, especially equations 54 and later. Following recombination, the important contributions to the evolution of the matter temperature are Compton cooling and adiabatic cooling.

The Compton cooling component accounts for the exchange of energy between electrons and the CMB photons. It is given by (Seager et al. 2000, their Equations 54 through 57):

$$(1) \quad \frac{dT_M}{dt} = \frac{8}{3} \frac{\sigma_T U n_e}{m_e c n_{\text{tot}}} (T_R - T_M)$$

Where  $T_M$  is the matter temperature,  $T_R$  is the radiation blackbody temperature,  $\sigma_T$  is the Thomson scattering cross section,  $U$  is the total energy density,  $n_e$  is the number density of electrons,  $m_e$  is the electron mass,  $c$  is the speed of light,  $n_{\text{tot}}$  is the number density of all species (electrons, hydrogen, and helium nuclei).

The adiabatic cooling is due to the expansion of the Universe and is given by (Seager et al. 2000, their Equation 58):

$$(2) \quad \frac{dT_M}{dt} = -2H(t)T_M$$

Using the relation between redshift and time (Seager et al. 2000, their Equation 65):

$$(3) \quad \frac{dT_M}{dz} = \frac{dt}{dz} \frac{dT_M}{dt}$$

We can add the above contributions for a single differential equation (Seager et al. 2000, their Equation 66, simplified to ignore the minor contributions):

$$(4) \quad (1+z) \frac{dT_M}{dz} = \frac{8\sigma_T U}{3H(z)m_e c} \frac{n_e}{n_e + n_H + n_{He}} \times (T_M - T_R) + 2T_M$$

Solving this equation will trace the matter temperature as a function of redshift. We note that the number density of hydrogen is  $n_H = \Omega_b \rho_{crit} (1+z)^3$ , where  $\rho_{crit} = 3 H_0^2 / (8 \pi G)$ . The Hubble constant is given by:

$$(5) \quad H = H_0 \sqrt{\Omega_M(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda}.$$

RECFAST and the other numerical codes calculate  $n_e$ ,  $n_H$ , and  $n_{He}$  self-consistently with a fully physical treatment. We will express the ionization fraction  $x_e$ , such that  $n_e = x_e * n_H$  and we use the helium fraction  $Y_p = 0.245$ . We set the radiation temperature  $T_R = T_{CMB_0} * (1+z)$ , where  $T_{CMB_0} = 2.725$  K. This is acceptable because the photons have a much higher heat capacity than the matter in this era and will not cool appreciably through the Compton interaction (Seager et al. 2000). Standard physical and cosmological constants are used for other parameters.

We can now recreate the basic RECFAST calculation, as well as explore changes to the matter temperature evolution due to different values of the residual ionization fraction,  $x_e$ , assumed after recombination. Figure 1 shows the full RECFAST calculation for  $x_e$  and  $T_M$ . We see in Figure 1 that the residual ionization fraction rapidly falls following recombination and quasi-asymptotes to  $x_e \sim \text{few} \times 10^{-4}$  below  $z < 500$ . The matter temperature decreases continually and is about 9.3 K at  $z=20$ , falling to 5.5 K by  $z=15$ .

## 2. Limiting Cases and Approximations to Equation (4)

When the first term (Compton cooling) in our Equation (4) dominates, the matter temperature simply follows the CMB temperature, directly proportional to  $1+z$ . When the second term (adiabatic cooling) in Equation (4) dominates, the matter temperature departs from the CMB temperature and follows the characteristic evolution of an adiabatically cooling ideal gas with temperature proportional to  $(1+z)^2$ . In the full treatment of Equation (4), the transition between the two cases occurs at  $z \sim 150$  (when  $T_R \sim T_M \sim 400$  K).

Various authors (including Pober et al.) use the  $(1+z)^2$  approximation for the evolution of the matter temperature and assign a transition redshift in order to get a specific profile. Popular choices of transition redshift seem to be  $z=200$  and  $z=150$ . However, I have seen no support for choosing  $z=200$ , whereas  $z=150$  is justified by the full RECFAST and other numerical code calculations.

## 3. Trial $x_e$ Cases

We now wish to investigate what effect changes to the asymptotic value of  $x_e$  have on the evolution of the matter temperature. To do so, we construct our own  $x_e(z)$  profiles in which the profile follows the RECFAST  $x_e$  solution between  $500 < z < 1000$ , but is forced to a constant  $x_e$  value below  $z < 500$ . We use

three trial forced values of  $x_e(z<500) = 10^{-5}$ ,  $10^{-4}$ , and  $10^{-3}$ . Figure 2 shows the matter temperature histories derived from these trial cases. It is clear that changing the asymptotic  $x_e$  tail can have a substantial effect of the matter temperature at  $z=20$ . By increasing or decreasing  $x_e$  by an order of magnitude compared to the RECFAST calculation, the matter temperature ranges between 3 to 15 K at  $z=20$ .

#### 4. Global 21cm Temperature

Now we wish to model the impact of our constructed matter temperature histories on the global 21cm measurement. The global 21cm absorption temperature is given by:

$$(6) \quad T_{21} = 28 * x_{\text{HI}} * \sqrt{(1+z)/10} * (1 - T_{\text{R}} / T_{\text{S}})$$

where  $x_{\text{HI}}$  is the neutral fraction (and is equal to  $1-x_e$ ) and  $T_{\text{S}}$  is the spin temperature of the hyperfine line. The spin temperature couples to both the CMB and the matter temperatures to varying degrees depending on the matter density and radiation conditions at a given epoch. Hence, in the scenarios we are considering,  $T_{\text{M}} \leq T_{\text{S}} \leq T_{\text{R}}$ . We will use the matter temperatures derived from the trial cases above and take  $T_{\text{S}} = T_{\text{M}}$ , which yields the most extreme absorption trough brightness temperatures. We neglect the details of the astrophysics that produce the actual trough shape, and restrict our attention only to the maximum possible magnitude of the trough. Figure 3 shows the maximum possible magnitude of the global 21cm absorption trough as a function of redshift for our trial cases.

#### 5. Constraining the IGM temperature with the Global 21cm Absorption Magnitude

The magnitude of an observed global 21cm absorption trough can be used to constrain the IGM (matter) temperature. During the First Light era that we are considering here, the matter temperature is always below the CMB temperature. We can see from Equation (6) above that the magnitude of the  $T_{21}$  signal diminishes to zero as  $T_{\text{S}} \rightarrow T_{\text{R}}$ , whereas it reaches a maximum magnitude when  $T_{\text{S}} = T_{\text{M}}$ . Therefore, the observed magnitude of a trough at a given redshift sets an upper bound on the IGM temperature at that redshift. Figure 4 shows the relationship between the observed 21cm trough amplitude and the upper limit on the IGM temperature for several redshifts.

#### 6. Discussion

Although we have treated  $x_e$  as a free parameter here, it should be noted that the outputs of CosmoRec and HyRec are considered to be very well established and accurate at the  $\sim 1$  percent level (see references, plus Planck results papers). In order to achieve a 500 mK global 21cm absorption trough feature at  $z=18$ , we need the matter temperature to be about half the temperature predicted by these codes. If this temperature difference is explained by a difference in  $x_e$ , then we need to alter the asymptotic  $x_e$  predictions of the codes by a factor of a few or more. Such a deviation from the code output is well beyond the currently accepted percent-level accuracy of the codes.

RECFAST-type codes assume homogeneity. Some deviations from the code results may be possible to due inhomogeneity as structure grows, although for small perturbations there are arguments that the effects would be minor or non-existent (R. Barkana, private communication)

Another mechanism that has been suggested that might help to cool the IGM (once the first stars appear) beyond the adiabatic level is higher Lyman line scattering leading to recombinations that inject photons into Lyman-alpha with essentially zero temperature. This mechanism has been considered by Chen & Miralda-Escude (2003), at end of their Section 2, and found to have very small contribution. (C. Hirata, private communication)

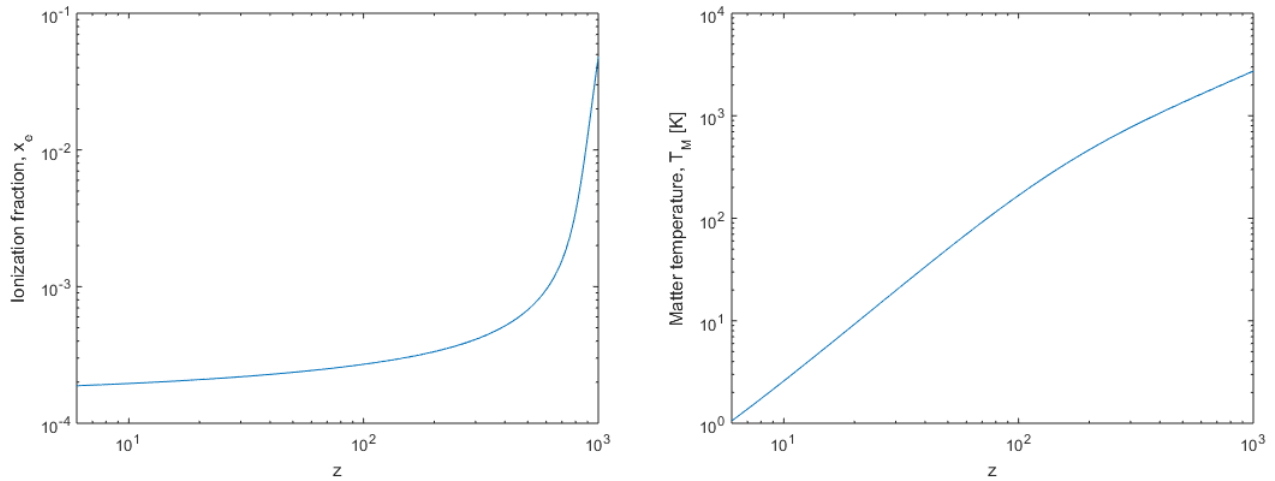


Figure 1. Ionization history (left) and matter temperature history (right) from the full RECFAST calculation. CosmoRec and HyRec yield small fractional changes to these curves at the order of  $\sim 1$  percent. Note that  $x_e$  quickly falls to a few  $\times 10^{-4}$  by  $z \sim 500$ . The matter temperature is,  $T_M = 9.3$  K at  $z=20$  and 5.5 K at  $z=15$ .

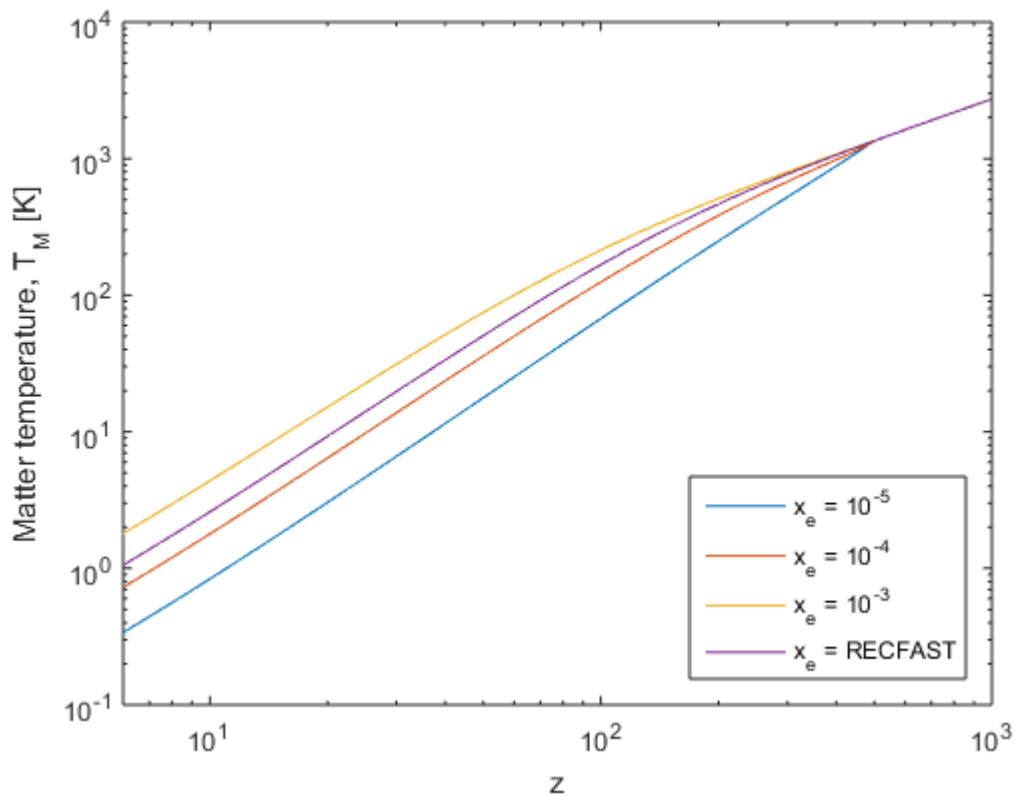


Figure 2. Examples of matter temperature histories for several forced values of ionization fraction. The fiducial temperature history from the full RECFAST calculation is also shown. For each of the forced  $x_e$  cases, the full RECFAST  $x_e$  history was used from  $500 < z < 1000$  and the forced constant value was used for  $z < 500$ . Our forced value replaces the full RECFAST solution in the redshift range where the full solution tends to asymptote to a nearly constant value of a few  $\times 10^{-4}$ . Note that lowering the asymptotic value of  $x_e$  can substantially lower the matter temperature below  $z < 100$ . The matter temperature at  $z=20$  is  $T_M = 3$  K, 6.4 K, and 15.1 K, respectively, for forced  $x_e = 10^{-5}$ ,  $10^{-4}$ , and  $10^{-3}$ .

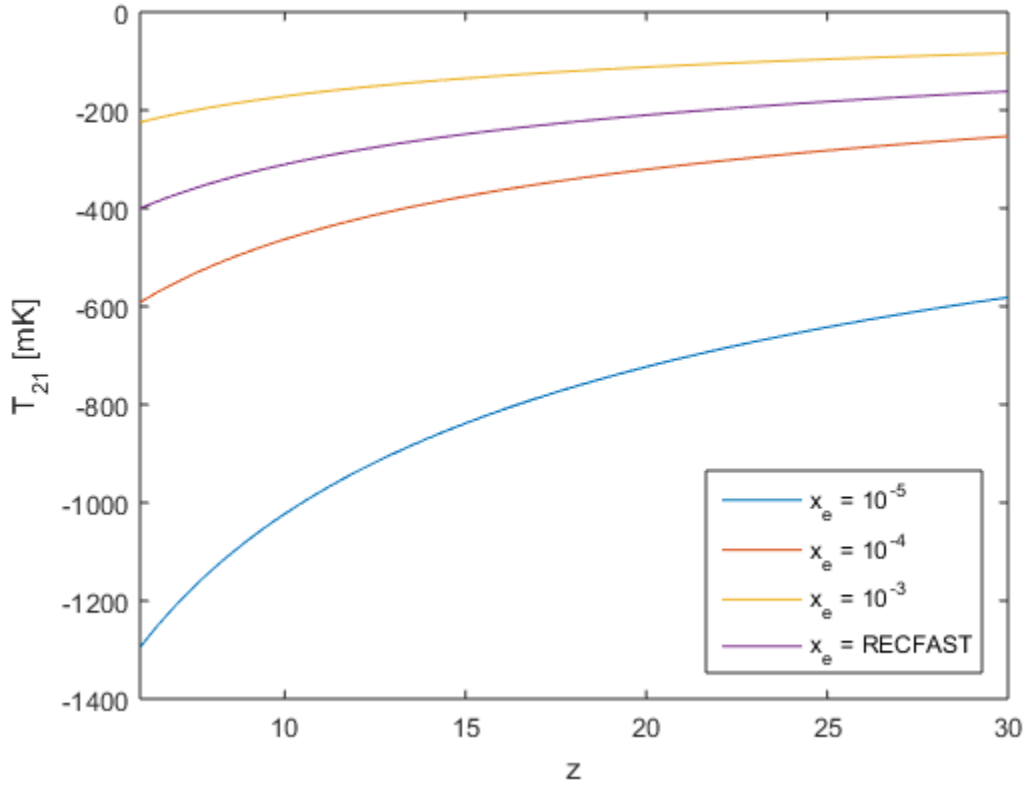


Figure 3. Examples of global 21cm absorption temperatures assuming fully saturated spin temperature (spin temperature equals the matter temperature). An asymptotic residual ionization fraction slightly below  $10^{-4}$  would be needed to account for a 500 mK absorption feature between  $15 < z < 20$ . At  $z=20$ , the maximum possible magnitude of the global 21cm absorption trough is  $T_{21} = -720$  mK,  $-320$  mK, and  $-112$  mK for  $x_e = 10^{-5}$ ,  $10^{-4}$ , and  $10^{-3}$ , respectively, while the full RECFAST calculations yield  $T_{21} = -210$  mK at  $z=20$ .

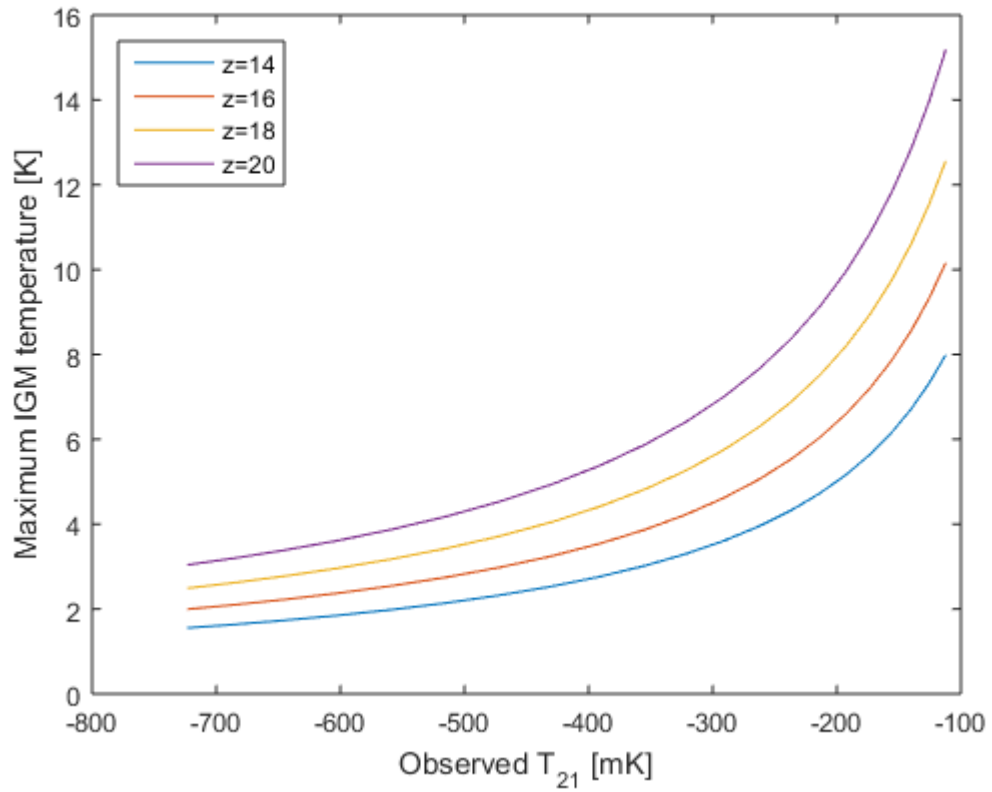


Figure 4. Upper limits on the IGM temperature plotted as the function of an observed global 21cm absorption trough amplitude. For an observed trough with absorption amplitude of 500 mK at  $z=18$ , the IGM temperature is constrained to be less than  $\sim 3.5$  K at that redshift.

## REFERENCES

1. Ali-Haïmoud, Yacine; Hirata, Christopher M., HyRec: A fast and highly accurate primordial hydrogen and helium recombination code, *Physical Review D*, vol. 83, Issue 4, id. 043513, 2011
2. Chen, Xuelei, Miralda-Escude, Jordi, THE SPIN-KINETIC TEMPERATURE COUPLING AND THE HEATING RATE DUE TO Ly-Alpha SCATTERING BEFORE REIONIZATION: PREDICTIONS FOR 21 CENTIMETER EMISSION AND ABSORPTION, *The Astrophysical Journal*, 602:1–11, 2004
3. Seager, Sara; Sasselov, Dimitar D.; Scott, Douglas, How Exactly Did the Universe Become Neutral?, *The Astrophysical Journal Supplement Series*, Volume 128, Issue 2, pp. 407-430, 2000
4. Shaw, J. R.; Chluba, J., Precise cosmological parameter estimation using COSMOREC, *Monthly Notices of the Royal Astronomical Society*, Volume 415, Issue 2, pp. 1343-1354, 2011