MWA Antenna Impedance Measurements

Hamdi Mani, Judd Bowman

***Abstract*—we describe measurements of the differential output impedance of the MWA bowtie antenna. The data shows that the magnitude of the impedance varies from 25 Ohm to 310 Ohm on the 80-280MHz band. The antenna has -5dB return loss relative to 200Ohm reference impedance from 150MHz to 380MHz, -5dB return loss relative to 100 Ohm reference impedance on the 100-390MHz band. Differential mode, common mode and mixed mode s-parameters are computed from the raw data. The obtained results agree quite well with the data taken at Haystack observatory. More work needs to be done to improve the impedance of the antenna**

***Index Terms*—radio telescope arrays, Dipole antennas, differential, common and mixed mode s-parameters, output impedance.**

I. Introduction

The MWA telescope or the Murchison Wide field Array is a large element count radio telescope array with up to 8192 antenna elements made of 512 tiles each is composed of 4 X 4 dipoles. Each tile is a phased array telescope that can be electronically steered without mechanically moving the array elements. This is done with analog beam former. Each antenna covers the frequency range: 80-300MHz band and is located on the radio quite zone on Western Australia within the Murchison observatory. The instrument is designed to achieve several science goals: detect the highly red shifted 21cm emission from the Epoch of re-ionization; this is in addition to conducting atmospheric science research and exploring the transient universe searching for radio transients [1].

II. The antenna

The antenna is a vertical crossed bowtie dipole; it has 2 linear orthogonal polarizations over ground screen with wide beam width and broad bandwidth. The dipoles are arranged in 16 element phased array (4 X 4) tiles spaced λ/2 at 140MHz (1.07m).

The dipole has a wide beam width centered at zenith with a conductive ground screen that causes the pattern to roll off at low elevation angles (<30 deg) reducing RFI pickup from terrestrial sources.

Hamdi Mani is a physics student at Arizona state university and works at the low frequency radio astronomy and experimental Cosmology Lab. He can be reached at: hamdi.mani@asu.edu

Judd Bowman is an assistant professor at ASU and is the group leader of the Low Freq radio astronomy and Cosmology group (Judd.Bowman@asu.edu)

The antenna is a key element of the array; it is the first element of the receiver chain and constitutes –with the LNA- the front end part of the telescope: the part of the system that will pick up and amplify the very weak radio signal. High sensitivity is a key requirement to achieve the stated science goals. From an engineering point of view, the high sensitivity requirement translates to a low receiver noise temperature.

The system noise (Tsys) at these low frequencies is dominated by sky noise mainly galactic radio background [2] (400K @150MHz) but a low receiver noise is preferable.

The total noise temperature of the receiver (Trx) (not including sky noise) is determined primarily by the noise temperature of the Low noise amplifier. The noise of the front end is determined by 4 factors as shown in Eq.1 and illustrated in the front end diagram in Fig1. :

1. The noise of the LNA itself: determined by noise of the transistors and the matching network.
2. The loss of the Antenna: determined by the resistivity of the material and the geometry (thickness and length): can be neglected for low frequency antennas.
3. The loss of the transmission line connecting the antenna to the LNA (resistivity of the conductor and length of the line): can be neglected for a very short line at low frequencies.
4. The output impedance of the antenna: determined by the antenna design and configuration.

Friis Equation summarizes all these factors:

$$T_{RX} = (L_{ant} - 1).T_{amb} + (L_{TL} - 1).T_{amb}.L_{TL}.L_{ant} + T_{LNA}.L_{TL}.L_{ant}$$

Eq.1

The output impedance of the antenna Zant is transformed to a different value by the transmission line, the new impedance Zs is considered the ‘generator’ or source impedance presented to the LNA. Zs is particularly important since the noise of the LNA is sensitive to the impedance at its input. This is better seen by looking at the 4 noise parameters of the LNA:

- Tmin: minimum achievable noise temperature.
- Zopt: Generator reflection coefficient that will give Tmin (Zopt= Ropt + i Xopt).
- N: noise ratio and Rn: noise resistance: this is a sensitivity parameter which determines how fast the

noise temperature degrades when the generator impedance is different from Zopt.

Eq.2 shows the noise parameters of a network and how they affect the input referred noise temperature. To get the minimum noise of the LNA, the antenna output impedance must be as close as possible to Zopt.

It is important to note that the 4 noise parameters are frequency dependent. The output impedance of the antenna also affects the transducer gain of the LNA and defines the shape of the gain curve.

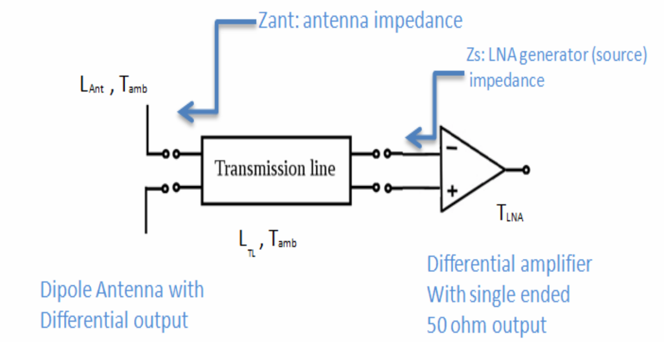


Fig1. MWA Front end showing the different discussed impedances: antenna output impedance, the LNA generator impedance and also shows the transmission line connecting the antenna to the LNA. The transmission line can add extra loss and also will change the antenna impedance Zant to different impedance (Zs); the latter affects the noise of the LNA. That is why it is important to measure Zs and not Zant. (in our remaining discussion, antenna impedance refers to Zs)

The output impedance of the antenna or the 'generator impedance' presented to the LNA is particularly important since the noise of the LNA is sensitive to the impedance at its input. This is better seen by looking at the noise parameters of the LNA. The LNA noise is minimum at particular input impedance called Zopt; to get the minimum noise the antenna output impedance must be as close as possible to Zopt.

$$T_n = T_{min} + T_0 . N . \frac{|Z_s - Z_{opt}|^2}{R_s - R_{opt}}$$

Eq.2

Where:

$$N = \frac{R_n}{R_{opt}}$$

And:

$$T_0 = 290K$$

The output impedance of the antenna also affects the transducer gain of the LNA and defines the shape of the gain curve.

$$G_T = \frac{1 - |\Gamma_s|^2}{|1 - \Gamma_{in}\Gamma_s|^2} |S_{21}|^2 \frac{1 - |\Gamma_L^2|^2}{|1 - S_{22}\Gamma_L|^2}$$

where Γ_{in}, the reflection coefficient looking into the input, is given by

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

Eq.3 and Eq.4

The output of the MWA dipole is differential, special instrument and techniques are needed to characterize this impedance.

III. Measurement Setup

The impedance measurements were done using the Agilent PNA-X (N5242) 4port network analyzer [3] at Caltech on 05/28/2010.

The antenna was measured on the roof of the building to avoid reflections as far as possible from the walls.

The VNA was calibrated using these parameters:

- Start Frequency: 10MHz
- Stop Frequency: 1000MHz
- Frequency Points: 991 point
- Test Port Power: -10dBm (for all 4 Ports)
- IF Bandwidth: 500Hz (Lower IF BW at lower frequencies)
- Averaging: 10 sweeps
- Calibration: SOLT using E-Cal Kit.
- Port Configuration: Logical Port1 (Balanced Port1): Port1 +Port3; Logical Port 2(Balanced Port 2): Port2 + Port4

Since the antenna does not have a coaxial interface at its output, it was necessary to mount an SMA connector of known electrical length to the twin line, this allowed direct connection to the network analyzer cables.

The network analyzer was calibrated at the end of its cables and a reference extension was applied to correct for the SMA connector: 50 ohm line, length=23mm in Air or 77ps delay measured by shorting the end of the SMA and adding an offset length till the impedance moved back to perfect short on the Smith chart. Fig2 shows the transmission line with SMA connectors.

Fig2. TL used for the impedance measurements. Also the home made cal kit used to remove the effect of the SMA connector.

This procedure moved the reference place to the end of the twin lines. This is exactly the impedance that the LNA is seeing.
Fig3.Shows the output of the feed with twin lines and SMA.

Fig3. The 2 polarizations of the antenna with SMA connectors ready to be measured.

To avoid reflections from the walls, the measurement instrument and the experimenter, the antenna was mounted on a plastic rod and lifted about 2meters off the floor.Fig4 shows the actual setup on the roof of the Moore building at Caltech

Fig4. Measurement setup on the west side of Caltech's Moore building showing the antenna far away from any wall or conductive objects.

IV. Differential, common mode and MIXED MODE S-parameters

Before discussing the measurement results, it is important to introduce the concepts of differential networks or networks with balanced ports and their s-parameter representations.

The antenna has a differential output; the signal from its output is transmitted using a pair of terminals (twin leads) where each terminal carries a voltage of equal amplitude and out of phase referenced to a virtual ground plane.

Here we define the 3 types of Ports/ excitation signals:

- Single ended signals: are applied to one input terminal of a single ended port relatively to the GND terminal
- Differential mode signals: are applied to both input ports of a balanced port and are of equal amplitude and opposite sign (180 deg phase difference). This is odd-mode propagation.
- Common mode signals: are applied to both input ports of a balanced port and are of equal amplitude and in phase (0 deg phase difference). This is even mode propagation.

Real world signals are not purely differential or common mode, they are a mix of both modes, in fact, every signal can be decomposed into a differential and common mode components. That is why real world signals are denoted mixed mode signals.

In balanced circuits, the differential mode signals are the wanted signals and the common mode signals are unwanted and are usually considered noise. That is why a good differential circuits process (amplify for example) the differential mode part of the input signal and reject the common mode part of it.

Some examples of common mode signals: radio frequency interference, coupled noise leaking out from a component of the system, power supply noise…
The balanced antenna can be modeled as a 1-port differential network which can be represented as a single ended 2-port network where the 2 ports are unbalanced. APPENDIX 1 shows a derivation of the conversion equations between single ended parameters and mixed mode parameters. The differential impedance measurement is reduced to measuring the single ended s parameters measurement and converting them to balanced, common mode and mixed mode s parameters.

The differential, mixed and common mode s-parameters are defined as:

$$S_{dd11} = \frac{b_d}{a_d} \qquad S_{dc11} = \frac{b_d}{a_c}$$
$$S_{cd11} = \frac{b_c}{a_d} \qquad S_{cc11} = \frac{b_c}{a_c} \qquad \text{Eq.5}$$

Where ad and ac are the incident differential and common mode normalized power waves and bd and bc are the reflected differential and common mode normalized power waves.

S_{dd11}: is the differential mode reflection coefficient, it describes the differential response of the port to a differential excitation wave. A low Sdd11 means a good impedance match. A high Sdd11 means the port is not well matched to the characteristic impedance causing reflections.

S_{dc11}: describes conversion of common mode waves to differential mode waves on port 1.

S_{cd11}: describes conversion of differential mode waves to common mode waves on port 1.
Non zero cross mode s-parameters means there is a conversion of signals between the different modes.
For a perfectly balanced circuit, the mixed mode s-parameters are zero.

S_{cc11}: is the common mode reflection coefficient, it describes the common mode response of the port to a common mode excitation signal. A high S_{cc11} means common mode signals incident on the port (noise, EMI, RFI…) are reflected and thus rejected. A low S_{cc11} means a large percentage of the incident common mode noise was delivered to the port and processed by the network.

The conversion is done manually on EDA software: AWR microwave office [4] and compared to the result given by the PNA-X network analyzer.

V. MEASUREMENT RESULTS

The single ended s-parameters of one polarization are recorded on a touchstone s2p file and then plotted on Fig5 on Magnitude dB format and on Fig6. on complex form on a smith chart.

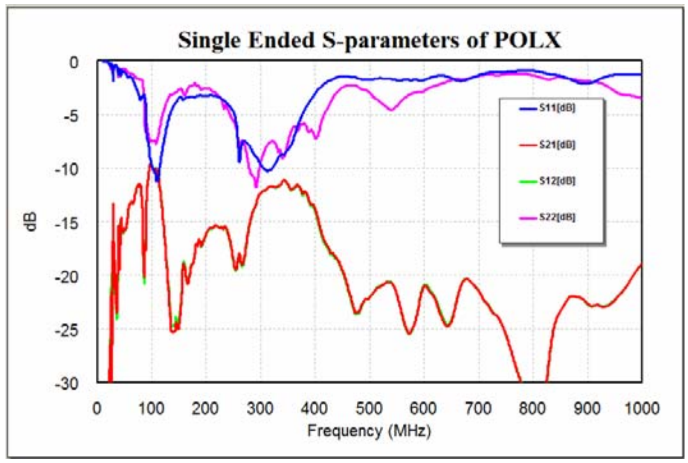


Fig5. Single-ended s-parameters of POLX Magnitude dB format. S21 shows the coupling between the 2 arms of the 1 polarization.

The balance between the 2 single ended ports looks good: S11 and S22 are very close; s12 and s21 are almost equal.

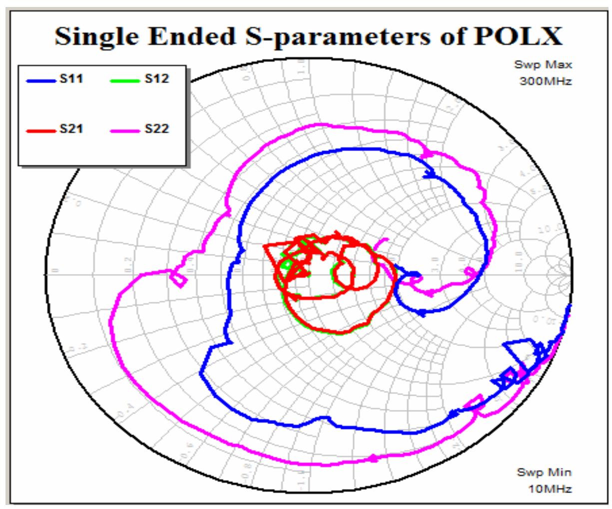


Fig6. Single-ended s-parameters of POLX in a smith chart Plot.

The differential, common mode and mixed mode s-parameters are then computed using equations derived on the APPENDIX and shown on Fig7.

DIFFERENTIAL MODE S PARAMETERS

Sdd11=0.5*(s11+s22-s21-s12)

COMMON MODE S PARAMETERS

Scc11=0.5*(s11+s12+s22+s21)

DIFFERENTIAL TO COMMON MODE CONVERSION S PARAMETERS

Scd11=0.5*(s11+s21-s12-s22)

COMMON TO DIFFERENTIAL MODE CONVERSION S PARAMETERS

Sdc11=0.5*(s11-s21+s12-s22)

Fig7. Single-ended to differential, common and mixed modes s-parameters conversion equations implemented in AWR MWO EDA software.

The 4 computed s parameters are shown on a Smith chart plot on Fig8 and on Log Magnitude plot on Fig9. A screen shot of the VNA measurements is shown on Fig10.
The same calculations are done for the other polarization, denoted by POLY and the complex differential impedances of the 2 polarizations are shown on the Smith Chart plot of Fig11 from 80MHz to 300MHz.
Fig11 shows that the impedance varies greatly on the 80-300MHz band.
For the sake of completeness, the differential impedances of both polarizations are plotted on Fig12 on the entire measurement band: 10MHz to 1GHz.

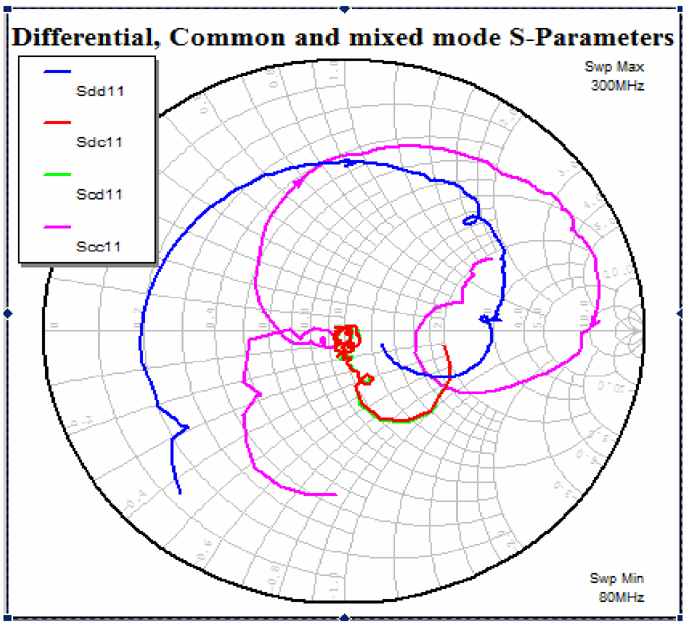


Fig8.Computed differential mode Sdd11, common mode Scc11 and mixed mode Sdc11 and Scd11 s-parameters on a Smith Chart.

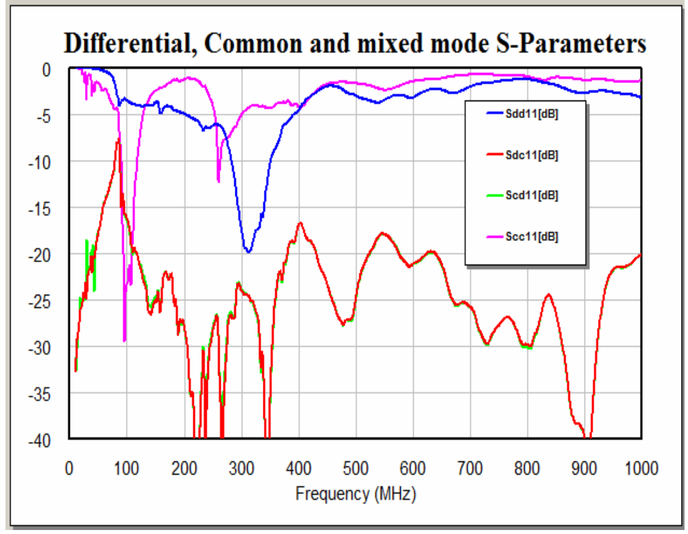


Fig9.Computed differential mode Sdd11, common mode Scc11 and mixed mode Sdc11 and Scd11 s-parameters on Log Magnitude chart. Sdd11 is referenced to 100 Ohm; Scc11 is referenced to 25 Ohm.

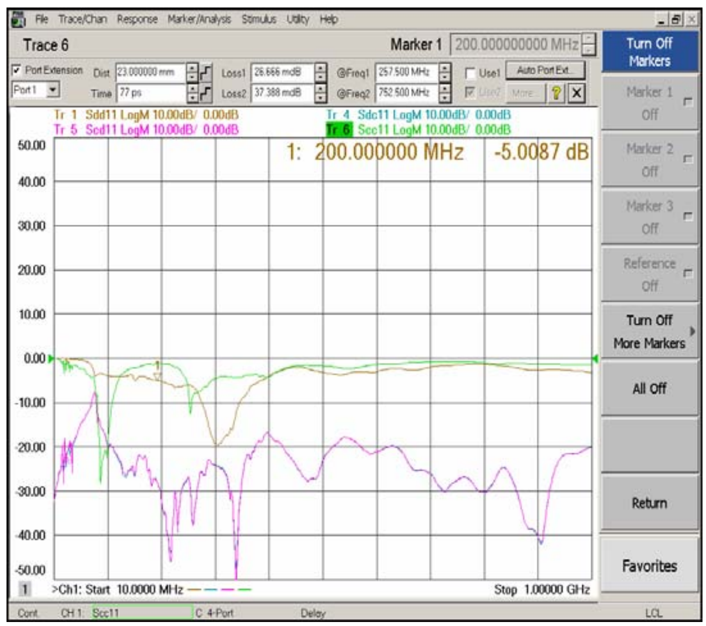


Fig10. Screen capture of the VNA while measuring the s-parameters and plotting the mixed mode parameters live during the measurement. The screen capture also shows the correction for the SMA connector, correction bas on measurement of an open and shorted SMA. Since the Z0=50 Ohm, The VNA assumes Sdd11 is referenced to 2.Z0=100 Ohm and Scc11 is referenced to 0.5.Z0=25 Ohm.

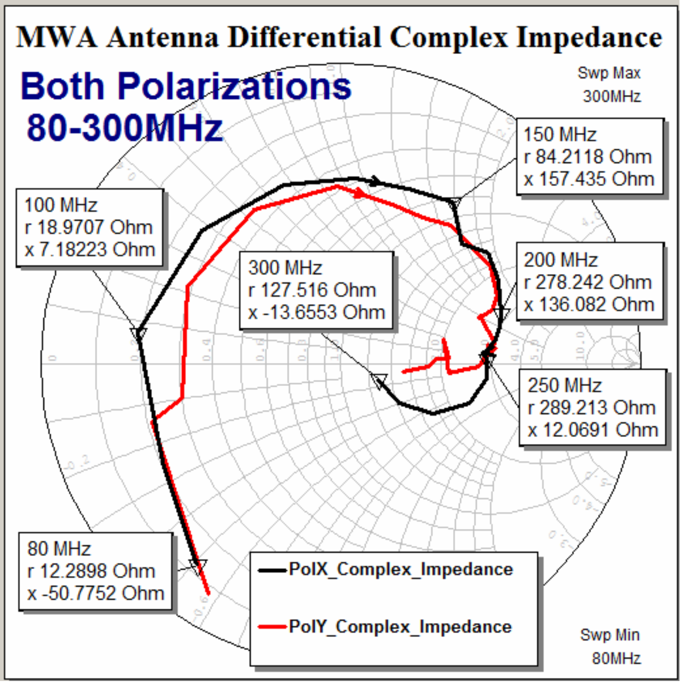


Fig11. The complex differential impedances of POLX and POLY are shown. The marker readings are impedance readings in real and imaginary format (reading are not normalized to any reference impedance). The impedance varies a lot on the band. The polarizations are quite similar due to good symmetry of the antenna.

The antenna has capacitive impedance on the 80-100MHz and 250-300MHz band and is inductive on the 100-250MHz band. The real part of the impedance varies from 12.2 Ohm at 80MHz to 127 Ohm at 300MHz.

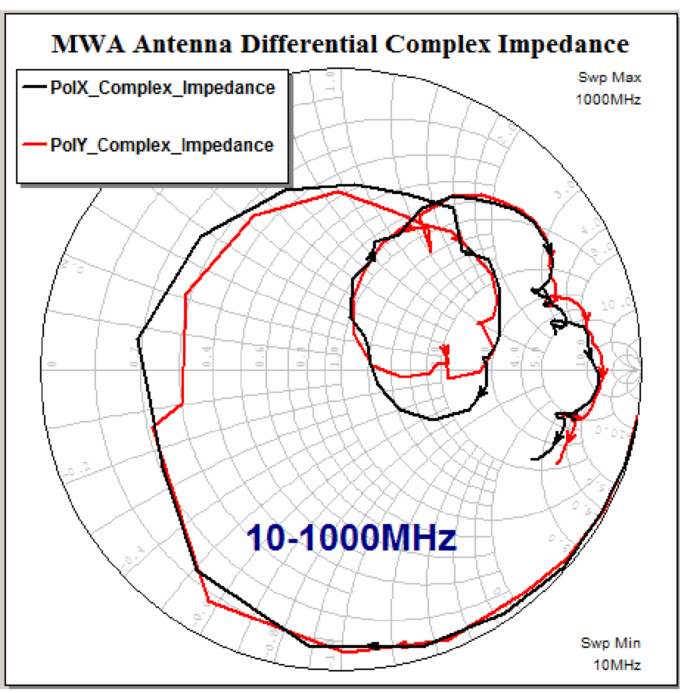


Fig12. The impedances are plotted on the entire measurement band.

Fig 13 shows the real and imaginary part of the impedance in a rectangular plot to compare the data measured with the described instrument and method to the data taken at MIT Haystack observatory by A. E.E. Rogers [5] (shown on Fig14), the scales are made the same to allow for easier comparison.

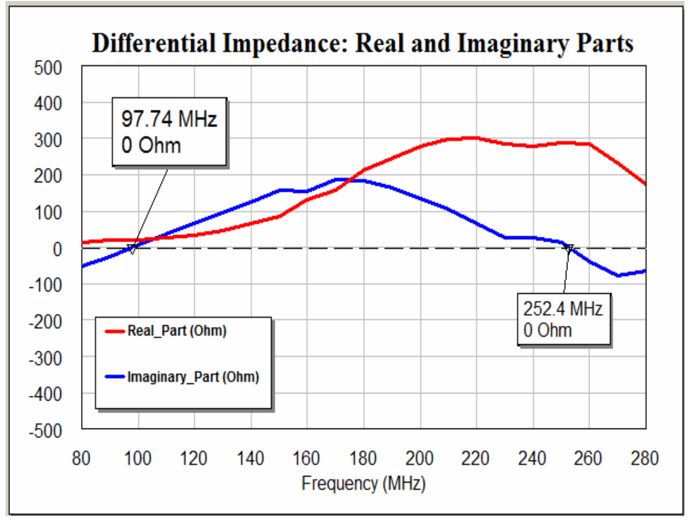


Fig13. Real and Imaginary part of the differential impedance measured by the author(impedance not normalized)

The data shows again the impedance variation over the band of interest, the antenna is resonant at 97MHz and 252MHz. The 2 data plots agree quite well despite the fact that 2 different instruments and methods have been used.

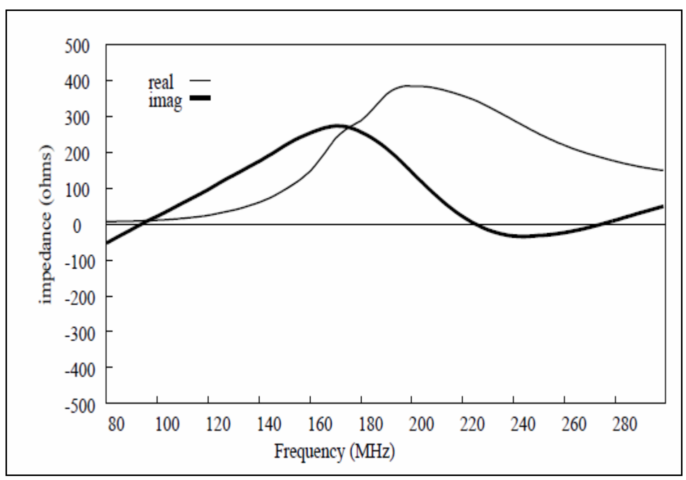


Fig14. Real and imaginary part of the impedance measured at Haystack observatory on one configuration of the twin lines. (A.E.E ROGERS, EDGES MEMO#31).

The return loss of the antenna relative to a characteristic impedance of: 100 Ohm, 150 Ohm and 200 Ohm are shown on Fig15, 16 and 17 respectively. These plots define the "impedance bandwidth" of the antenna for an intended use as 100, 200 or 270 Ohm antenna.

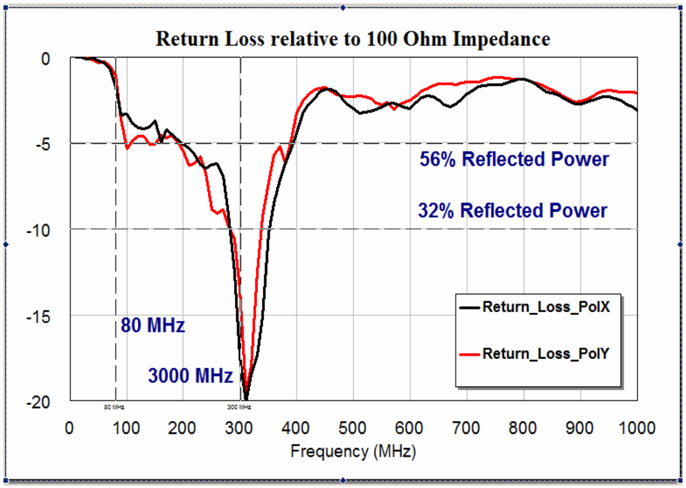


Fig15.The return loss [dB]of the antenna relative to 100 Ohm characteristic impedance.

The return loss is defined as: RL=20*log10 (Γ), where Γ is the voltage reflection coefficient relative to the characteristic impedance. The antenna has 5dB return loss on the 1000-400MHz band which means that 56% of the power available from the antenna terminals will be reflected back to the antenna when it is connected to a 100 Ohm (differential) impedance load (the differential input impedance of the LNA for example), only 44% of the available power gets delivered to the rest of the system.

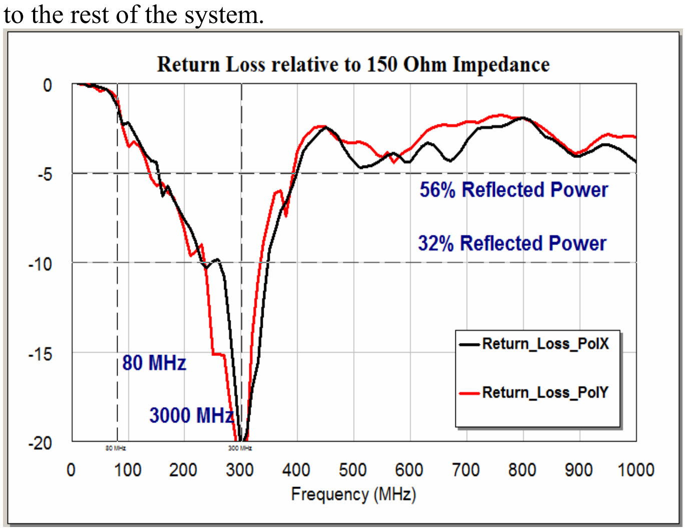


Fig16.The return loss[dB] of the antenna relative to 150 Ohm characteristic impedance.

The impedance bandwidth in this case is 150-400MHz assuming that a 5dB return loss (56% reflected power) relative to 150 Ohm is minimum.

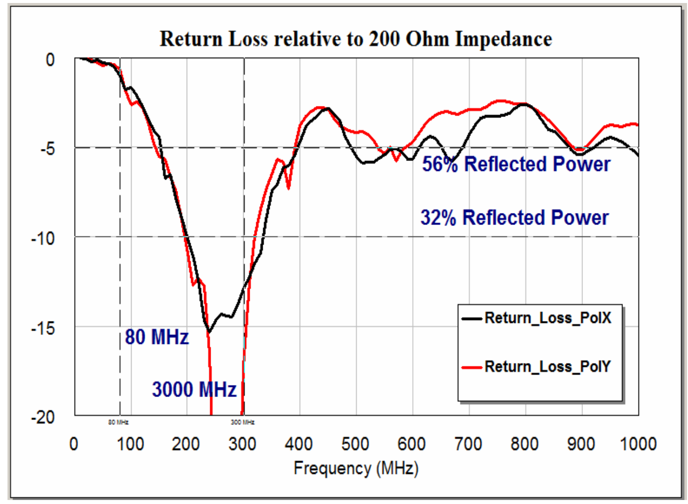


Fig17.The return loss[dB] of the antenna relative to 200 Ohm characteristic impedance.

The impedance match has degraded at lower frequencies when the return loss referenced to 200 Ohm characteristic impedance is plotted.

The antenna has a 10dB return loss (68% power transfer from the antenna to the remaining system) on the 200-340MHz band.

Fig18 is a plot of the mixed mode (or mode conversion) s-parameters: Scd11 and Sdc11.

The non zeros Sdc11 tells that the antenna is not perfectly balanced and that some of the common mode waves incident on port1 are reflected back as differential mode signals.

In most of the band (except at 85MHz) , the conversion is -20dB or 10% of the incident common mode signal (RFI, EMI, noise…) incident on the antenna port is reflected back as differential mode signal and gets processed by the succeeding differential LNA for example. The total energy is conserved if we neglect ohmic losses.

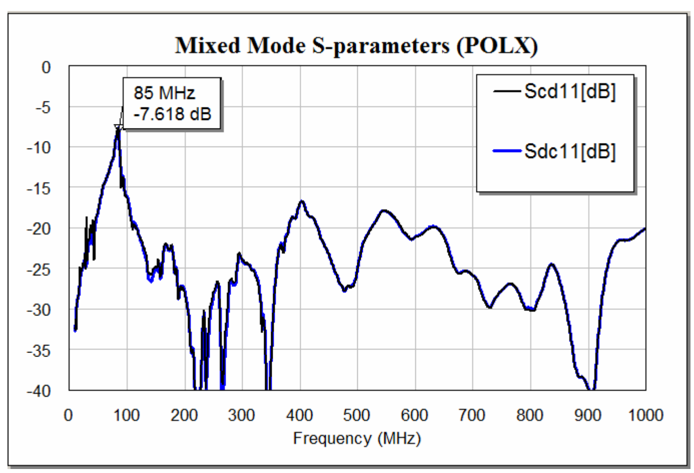


Fig18.Cross mode s-parameters. Scd11 and Sdc11 [dB]are equal which means that the conversions between the modes are equal.

Similarly for Scd11: 10% of the incident differential mode signal (reflected by a poorly matched differential LNA for example) is reflected back (to the LNA in this case) as a common mode signal that gets processed(amplified) by the differential LNA in case the differential LNA has a poor common mode rejection ratio.(in case the LNA has a high common mode rejection ratio, the common mode signals get reflected or attenuated).

The magnitude of the common mode reflection coefficient S_{cc11} is shown on Fig19.

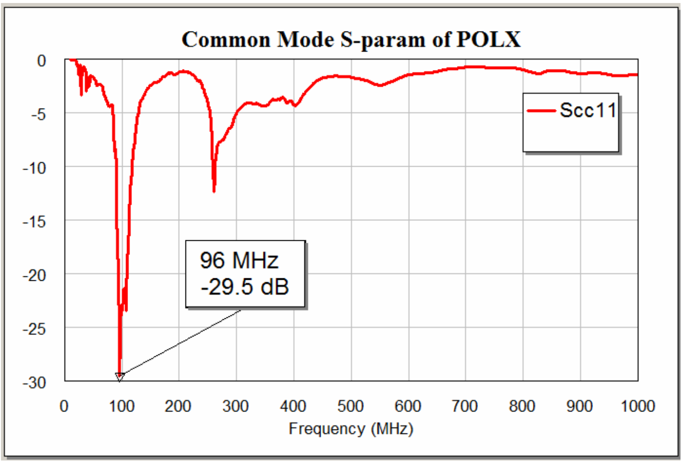
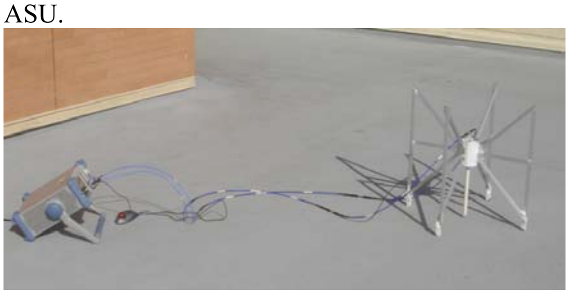


Fig19.Common mode s-parameters. Scc11[dB] indicates poor common mode return loss.

S_{cc11} is higher than -5dB on most of the band (except at 96 MHz) which means that most of the common mode signals incident on the antenna port are reflected (>56%) thus rejected.

VI. IMPEDANCE MEASUREMENTS AT ASU

All the plots shown so far are based on data taken at Caltech using a 4 port analyzer.
The same antenna was measured at ASU on the roof of the PSF building as far as possible from the walls to avoid reflections.
Since the VNA does not compute the differential sparam, post processing of the 2 port s2p file was done in Microwave office.
Fig20 shows the setup and the connection to the antenna.
Fig21. Shows a plot of the differential reflection coefficient (log10 mag relative to 100 Ohm) measured at Caltech and ASU.



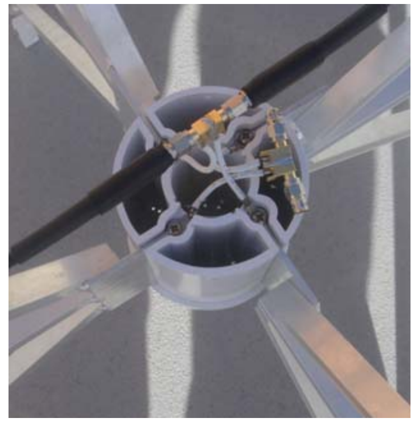


Fig20. Top: the VNA was placed as far as cable length allows from the antenna.Bottom: connection to one polarization of the antenna while the second polarization is terminated in 100 Ohm (2X50 ohm terminations)

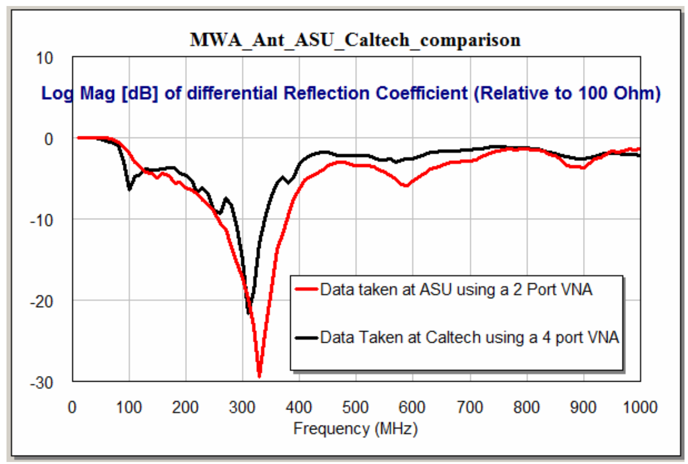


Fig21. Antenna return loss data measured at ASU and Caltech.Good agreement is seen

VII. CONCLUSION & DISCUSSION

The impedance of the MWA antenna varies a lot on the bandwidth of interest, the will affect the noise and the gain of the LNA.
More measurements need to be done:

1. Estimate the effect of the twin lead transmission line connecting the output of the antenna to the rest of the circuit. If the impedance is found to vary with the transmission line configuration (shape), work on a more robust and repeatable connection.
2. Measure the impedance of the antenna using a mini circuits 3dB 180deg coupler and correct for the loss of the coupler. Compare the 2 methods.
3. Re-measure the antenna impedance with a ground shield; this can be done on the roof of PSF at ASU using the R&S ZVL3 portable VNA.
4. Measure the impedance of the antenna while installed on the tile, this can be done on the field using the portable VNA, use trace math to compute the

differential impedance from the single ended s-parameters since the portable instrument is a 2-port VNA.

5. try to improve the impedance of the antenna using computer EM simulations.
6. try to improve the antenna impedance on the desired bandwidth by designing a low loss matching network between the antenna and LNA.
7. Measure the coupling between the 2 polarizations and the coupling between 2 antennas on the tile(different combinations)
8. Investigate the issues related to noise coming out of an antenna from a tile coupling to another antenna on the same tile and causing all sorts of issues.(Look at the SKA phase array feed reports)
9. Build a setup to measure the pattern of the antenna on the roof of the building at ASU, the setup can be similar to the one used at Caltech to measure higher frequencies antennas. the setup can be a used to measure antennas on the field too and confirm models: all what is required is:
   a. A stepper motor with controller that can be remotely programmed by MATLAB
   b. A transmit antenna covering the band.
   c. Remotely program the portable network analyzer using a MATLAB installed on the ZVL VNA
10. Simulate the noise of the LNA when terminated with the impedance of the antenna and measure the total noise of the antenna and LNA.

VIII. ACCOMPANYING FILES

- MWA_Antenna_Impedance_Data_Caltech_052010: Microwave office file containing all raw and processed data with plots and circuit diagrams.
- POLX_2port_SingleEnded.s2p : touchstone file containing the single ended 2-port s-parameters of POLX
- POLX_2port_SingleEnded.s2p: polY sep file
- MWA_POLX_Sdd_Sdc_Scd_Scc_05282010_Data.xlsx : excel file containing the mixed mode s parameters computed by the Agilent PNAx VNAfor POLX
- MWA_POLY_Sdd_Sdc_Scd_Scc_05282010_Data.xlsx : data for POLY

AKNOLWEDGEMENT

The author likes to thank graduate student Zan Zhang for his help setting up the measurement on the roof of the lab.

REFERENCES

[1] http://mwatelescope.com/
[2] ITU **P.372 : Radio noise**
[3] http://www.home.agilent.com/agilent/product.jspx?nid=-536902643.426031.00&cc=US&lc=eng
[4] www.**awrcorp**.com
[5] Alan E.E. Rodgers, EDGES Memo #31, June 2007

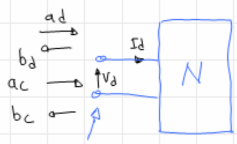
APPENDIX

APPENDIX: SINGLE ENDED TO MIXED MODE S-PARAMETERS CONVERSION EQUATIONS DERIVATION

• DIFFERENTIAL 1-PORT NETWORS REPRESENTED AS A 2-PORT SINGLE ENDED NETWORK

12/23/2010

BALANCED 1- PORT NETWORK



2-SIGNAL TERMINALS (PORT 1)

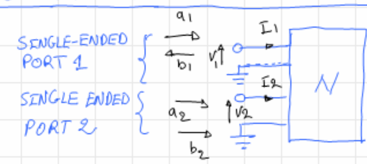
$[b] = [S] . [a]$

$$\begin{bmatrix} b_d \\ b_c \end{bmatrix} = \begin{bmatrix} S_{dd11} & S_{dc11} \\ S_{cd11} & S_{cc11} \end{bmatrix} . \begin{bmatrix} a_d \\ a_c \end{bmatrix}$$

$$\boxed{\begin{aligned} b_{d1} &= S_{dd11} . a_{d1} + S_{dc11} . a_{c1} \\ b_{c1} &= S_{cd11} . a_{d1} + S_{cc11} . a_{c1} \end{aligned}}$$

$\equiv$

SINGLE-ENDED 2-PORT NETWORKS



$[b] = [S] . [a]$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} . \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$\boxed{\begin{aligned} b_1 &= S_{11} . a_1 + S_{12} . a_2 \\ b_2 &= S_{21} . a_1 + S_{22} . a_2 \end{aligned}}$$

• a_1 AND a_2 ARE THE NORMALIZED INCIDENT POWER WAVES ON PORT 1 AND PORT 2 RESPECTIVELY:

$$\boxed{a_1 = \frac{1}{2\sqrt{Z_0}} (V_1 + Z_0 I_1)} \quad \boxed{a_2 = \frac{1}{2\sqrt{Z_0}} (V_2 + Z_0 I_2)}$$

• b_1 AND b_2 ARE THE NORMALIZED REFLECTED POWER WAVES FROM PORT 1 AND 2 RESPECTIVELY:

$$\boxed{b_1 = \frac{1}{2\sqrt{Z_0}} (V_1 - Z_0 . I_1)}$$

$$\boxed{b_2 = \frac{1}{2\sqrt{Z_0}} (V_2 - Z_0 . I_2)}$$

REFERENCE: "POWER WAVES AND THE SCATTERING MATRIX" K. KUROKAWA

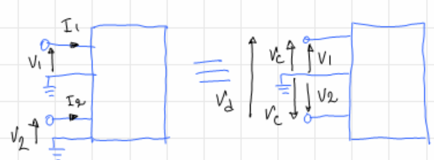
WE NEED TO FIND AN EXPRESSION OF a_d, a_c, b_d AND b_c AS A FUNCTION OF a_1, b_1, a_2 AND b_2

a_d: NORMALIZED DIFFERENTIAL MODE INCIDENT POWER WAVE: $\boxed{a_d = \frac{1}{2\sqrt{Z_{d_0}}} (V_d + Z_{d_0} . I_d)}$

a_c: NORMALIZED COMMON MODE INCIDENT POWER WAVE: $\boxed{a_c = \frac{1}{2\sqrt{Z_{c_0}}} (V_c + Z_{c_0} . I_c)}$

b_d: NORMALIZED DIFFERENTIAL MODE REFLECTED POWE WAVE: $\boxed{b_d = \frac{1}{2\sqrt{Z_{d_0}}} (V_d - Z_{d_0} . I_d)}$

b_c: NORMALIZED COMMON MODE REFLECTED POWER WAVE: $\boxed{b_c = \frac{1}{2\sqrt{Z_{c_0}}} (V_c - Z_{c_0} . I_c)}$



• THE DIFFERENTIAL MODE VOLTAGE BETWEEN THE 2 TERMINALS OF A BALANCES PORT IS DEFINED AS $\boxed{V_d = V_1 - V_2}$

• THE DIFFERENTIAL MODE CURRENT IS DEFINED AS: $\boxed{I_d = \frac{1}{2} (I_1 - I_2)}$

• THE DIFFERENTIAL IMPEDANCE IS THEN:

$$Z_{d_0} = \frac{V_d}{I_d} = 2 . \frac{V_1 - V_2}{I_1 - I_2} = 2 . Z_0 \Rightarrow \boxed{Z_{d_0} = 2 . Z_0}$$

$$\frac{V_1}{I_1} = \frac{V_2}{I_2} = Z_0 \rightarrow V_1 . I_2 = V_2 . I_1$$

$$\rightarrow \frac{I_2}{I_1} = \frac{V_2}{V_1}$$

$$\frac{V_1 - V_2}{I_1 - I_2} = \frac{V_1 - \frac{V_1 . I_2}{I_1}}{I_1 - \frac{V_2 . I_1}{V_1}} = \frac{V_1 . \left(1 - \frac{I_2}{I_1}\right)}{I_1 . \left(1 - \frac{V_2}{V_1}\right)} = \frac{V_1}{I_1} = Z_0$$

• THE COMMON MODE VOLTAGE ACROSS THE 2 TERMINALS IS DEFINED AS: $\boxed{V_c = \frac{1}{2} (V_1 + V_2)}$

• THE COMMON MODE CURRENT: $\boxed{I_c = I_1 + I_2}$

• THE COMMON MODE IMPEDANCE: $\boxed{Z_{c_0} = \frac{V_c}{I_c} = \frac{Z_0}{2}}$

$$\left(\frac{V_2 + V_1}{I_2 + I_1} = \frac{V_2 + V_2 . I_1/I_2}{I_2 + V_1 . I_2/I_1} = Z_0 \right)$$

$$a_d = \frac{1}{2\sqrt{Z_{d_0}}}\left(V_d + Z_{d_0}\cdot I_d\right)$$
$$= \frac{1}{2\sqrt{2Z_0}}\left(V_1 - V_2 + 2Z_0\cdot\frac{1}{2}(I_1 - I_2)\right)$$
$$= \frac{1}{2\cdot\sqrt{2}\cdot\sqrt{Z_0}}\cdot\left(V_1 - V_2 + Z_0 I_1 - Z_0\cdot I_2\right)$$
$$= \frac{1}{2\cdot\sqrt{2}\cdot\sqrt{Z_0}}\left[V_1 + Z_0\cdot I_1 - \left(V_2 + Z_0\cdot I_2\right)\right]$$
$$= \frac{1}{\sqrt{2}}\cdot\left[\frac{1}{2\sqrt{Z_0}}\left(V_1 + Z_0\cdot I_1\right) - \frac{1}{2\sqrt{Z_0}}\left(V_2 + Z_0\cdot I_2\right)\right]$$
$$= \frac{1}{\sqrt{2}}\left(a_1 - a_2\right)$$

$$b_d = \frac{1}{2\sqrt{Z_{d_0}}}\left(V_d - Z_{d_0} I_d\right)$$
$$= \frac{1}{2\cdot\sqrt{2}\cdot\sqrt{Z_0}}\left(V_1 - V_2 - \cancel{2}Z_0\,\frac{\cancel{1}}{\cancel{2}}(I_1 - I_2)\right)$$
$$= \frac{1}{\sqrt{2}}\left[\frac{1}{2\sqrt{Z_0}}\left(V_1 - Z_0\cdot I_1\right) - \frac{1}{2\sqrt{Z_0}}\left(V_2 - Z_0\cdot I_2\right)\right]$$

$$b_d = \frac{1}{\sqrt{2}}\left(b_1 - b_2\right)$$

$$a_c = \frac{1}{2\sqrt{Z_{c_0}}}\left(V_c + Z_{c_0} I_c\right)$$
$$= \frac{1}{2\cdot\sqrt{Z_0/2}}\left(\frac{V_1 + V_2}{2} + \frac{Z_0}{2}(I_1 + I_2)\right)$$
$$= \frac{\sqrt{2}}{2\sqrt{Z_0}}\cdot\frac{1}{2}\left(V_1 + Z_0 I_1 + \left(V_2 + Z_0\cdot I_2\right)\right)$$
$$= \frac{1}{\sqrt{2}}\left(a_1 + a_2\right)$$

$$b_c = \frac{1}{2\sqrt{Z_{c_0}}}\left(V_c - Z_{c_0} I_c\right) = \frac{\sqrt{2}}{2\sqrt{Z_0}}\cdot\frac{1}{2}\cdot\left(V_1 + V_2 - Z_0(I_1 + I_2)\right)$$
$$= \frac{1}{\sqrt{2}}\cdot\frac{1}{2\sqrt{Z_0}}\left[V_1 - Z_0 I_1 + \left(V_2 - Z_0 I_2\right)\right]$$
$$= \frac{1}{\sqrt{2}}\cdot\left(b_1 + b_2\right)$$

$$\Rightarrow \boxed{\begin{array}{l} a_d = \frac{1}{\sqrt{2}}\cdot(a_1 - a_2)\ ;\ b_d = \frac{1}{\sqrt{2}}(b_1 - b_2) \\ a_c = \frac{1}{\sqrt{2}}(a_1 + a_2)\ ;\ b_c = \frac{1}{\sqrt{2}}(b_1 + b_2) \end{array}}$$

$$a_d + a_c = \frac{2}{\sqrt{2}}a_1 \Rightarrow a_1 = \frac{1}{\sqrt{2}}(a_d + a_c)$$
$$a_c - a_d = \frac{1}{\sqrt{2}}(a_1 + a_2 - a_1 + a_2) = \sqrt{2}\cdot a_2$$
$$a_2 = \frac{1}{\sqrt{2}}(a_c - a_d)$$
$$b_d + b_c = \frac{2}{\sqrt{2}}(b_1) = \sqrt{2}\,b_1 \rightarrow b_1 = \frac{1}{\sqrt{2}}(b_d + b_c)$$
$$b_c - b_d = \frac{1}{\sqrt{2}}(b_1 + b_2 - b_1 + b_2) = \sqrt{2}\cdot b_2$$
$$b_2 = \frac{1}{\sqrt{2}}(b_c - b_d)$$

$$\Rightarrow \boxed{\begin{array}{l} a_1 = \frac{1}{\sqrt{2}}(a_d + a_c)\ ;\ b_1 = \frac{1}{\sqrt{2}}(b_d + b_c) \\ a_2 = \frac{1}{\sqrt{2}}(a_c - a_d)\ ;\ b_2 = \frac{1}{\sqrt{2}}(b_c - b_d) \end{array}}$$

$$b_1 = S_{11}\cdot a_1 + S_{12}\cdot a_2$$
$$\cancel{\frac{1}{\sqrt{2}}}(b_d + b_c) = S_{11}\cdot\cancel{\frac{1}{\sqrt{2}}}(a_d + a_c) + S_{12}\cdot\cancel{\frac{1}{\sqrt{2}}}(a_c - a_d)$$
$$b_d + b_c = (S_{11} - S_{12})\cdot a_d + (S_{11} + S_{12})\cdot a_c \quad (Eq1)$$
$$b_2 = S_{21}\cdot a_1 + S_{22}\cdot a_2$$
$$\cancel{\frac{1}{\sqrt{2}}}(b_c - b_d) = S_{21}\cdot\cancel{\frac{1}{\sqrt{2}}}(a_d + a_c) + S_{22}\cdot\cancel{\frac{1}{\sqrt{2}}}(a_c - a_d)$$
$$b_c - b_d = (S_{21} - S_{22})\cdot a_d + (S_{21} + S_{22})\cdot a_c \quad (Eq2)$$

$$(Eq1) + (Eq2) \Rightarrow 2.b_c = (S_{11} + S_{21} - S_{12} - S_{22})a_d + (S_{11} + S_{12} + S_{21} + S_{22})\,a_c$$

$$\begin{cases} b_c = \frac{1}{2}(S_{11} + S_{21} - S_{12} - S_{22})\cdot a_d + \frac{1}{2}(S_{11} + S_{12} + S_{21} + S_{22}) \times a_c \\ b_{c_1} = S_{cd_{11}}\cdot a_{d_1} + S_{cc_{11}}\cdot a_{c_1} \end{cases}$$

$$\Rightarrow \begin{cases} S_{cd_{11}} = \frac{1}{2}(S_{11} + S_{21} - S_{12} - S_{22}) \\ S_{cc_{11}} = \frac{1}{2}(S_{11} + S_{22} + S_{12} + S_{21}) \end{cases}$$

$$\cdot\,(Eq1) - (Eq2) \Rightarrow 2\,b_d = \left(S_{11} - S_{12} - S_{21} + S_{22}\right)\cdot a_d + \left(S_{11} + S_{12} - S_{21} - S_{22}\right)\cdot a_c$$

$$\Rightarrow \begin{cases} b_d = \frac{1}{2}\left(S_{11} - S_{12} + S_{21} + S_{22}\right)\cdot a_d + \frac{1}{2}\left(S_{11} + S_{12} - S_{21} - S_{22}\right)\cdot a_c \\ b_d = S_{dd_{11}}\cdot a_d + S_{dc_{11}}\cdot a_c \end{cases} \Rightarrow \begin{cases} S_{dd_{11}} = \frac{1}{2}\left(S_{11} + S_{22} - S_{21} - S_{12}\right) \\ S_{dc_{11}} = \frac{1}{2}\left(S_{11} + S_{12} - S_{21} - S_{22}\right) \end{cases}$$

- <u>SINGLE ENDED TO DIFFERENTIAL MODE S-PARAMETERS CONVERSION EQUATIONS FOR A 2-PORT SINGLE ENDED NETWORK TO ITS EQUIVALENT 1-PORT DIFFERENTIAL NETWORK</u>:

$$S_{dd_{11}} = \frac{1}{2}\left(S_{11} + S_{22} - S_{21} - S_{12}\right)$$
$$S_{dc_{11}} = \frac{1}{2}\left(S_{11} + S_{12} - S_{21} - S_{22}\right)$$
$$S_{cd_{11}} = \frac{1}{2}\left(S_{11} + S_{21} - S_{22} - S_{12}\right)$$
$$S_{cc_{11}} = \frac{1}{2}\left(S_{11} + S_{22} + S_{12} + S_{21}\right)$$

- S_{11} AND S_{22} ARE INPUT/OUTPUT REFLECTION COEFFICIENTS OF SINGLE ENDED PORTS 1 AND 2 REFERENCED TO Z0: $\begin{cases} S_{11} = \Gamma_{in} = \frac{Z_{IN} - Z_0}{Z_{IN} + Z_0} \\ S_{22} = \Gamma_{OUT} = \frac{Z_{OUT} - Z}{Z_{OUT} + Z_0} \end{cases}$
- $S_{dd_{11}}$ IS THE DIFF REF COEF RELATIVE TO $Z_{d_0} = 2\,Z_0$
- $S_{cc_{11}}$ IS THE COMMON MODE REF COEF RELATIVE TO $Z_{c0} = \frac{Z_0}{2}$

<u>REFERENCE:</u> "PURE-MODE NETWORK ANALYZER FOR ON-WAFER MEASUREMENTS OF MIXED-MODE S-PARAMETERS OF DIFFERENTIAL CIRCUITS"
D.E. BOCKELMAN AND W. R. EISENSTADT
IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES. VOL 45. NO 7. JULY 1997